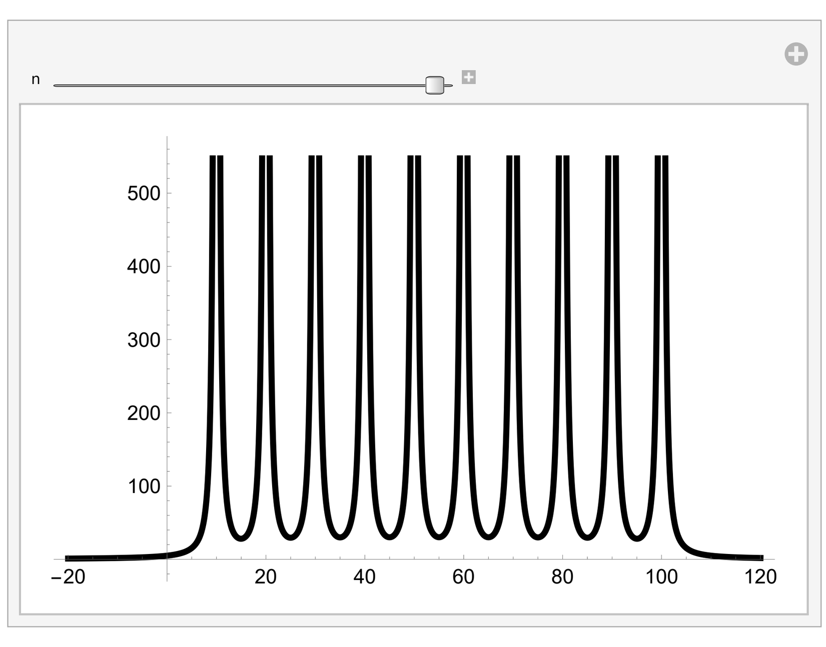
Firstly, let's see the potential that we are dealing with:



Starting from T.I. Schrodinger's Equation

$$\Psi''(x) = -\frac{2\, m\, \Psi(x)\, (T - \alpha\, \delta_x(x - x_n))}{\hbar^2}$$

$$\int_{-\epsilon}^{\epsilon}\left(\Psi''(x) = -\frac{2\, m\, \Psi(x)\, (T - \alpha\, \delta_x(x - x_n))}{\hbar^2}\right) dx$$

$$\Psi'(\epsilon) - \Psi'(-\epsilon) = \frac{2\, \alpha\, m\, \Psi(x_n)}{\hbar^2}$$

$$\lim_{\epsilon \to x_n} (\Psi'(\epsilon) - \Psi'(-\epsilon)) = \Psi_+(x_n) - \Psi_-(x_n)$$

$$\Psi_+'(x_n) - \Psi_-'(x_n) = \frac{2\, \alpha\, m\, \Psi(x_n)}{\hbar^2}$$

So we have two boundary conditions, The first is:

$$\Psi_+(x_n) = \Psi_-(x_n)$$

$$A_n\, e^{i k x} + B_n\, e^{-i k x} = A_{n+1}\, e^{i k x} + B_{n+1}\, e^{-i k x}$$

The second:

$$\Psi_+'(x_n) - \Psi_-'(x_n) = \frac{2\, m\, \Psi(x_n)}{\hbar^2}$$

$$\Psi_-'(x_n) - \Psi_+'(x_n) = \frac{2\, \alpha\, m \left(A_n\, e^{i k x_{nn}} + B_n\, e^{-i k x_{nn}}\right)}{\hbar^2}$$

$$i\, k\, A_n\, e^{i k x_n} - i\, k\, B_n\, e^{-i k x_n} = i\, k\, A_{n+1}\, e^{i k x_n} - i\, k\, B_{n+1}\, e^{-i k x_n} = \frac{2\, \alpha\, m \left(A_n\, e^{i k x_{nn}} + B_n\, e^{-i k x_{nn}}\right)}{\hbar^2}$$

Calling $\beta = \frac{m\, \alpha}{\hbar^2\, k}$, Then simplifying the last equation

$$-i\, k\, e^{-i k x_n} \left(A_n\, e^{2 i k x_n} - A_{n+1}\, e^{2 i k x_n} - B_n + B_{n+1}\right) = \frac{2\, \alpha\, m\, e^{-i k x_{nn}} \left(B_n + A_n\, e^{2 i k x_{nn}}\right)}{\hbar^2}$$

$$e^{-i k x_n} \left(-A_n\, e^{2 i k x_n} + B_n + F\, e^{2 i k x_n} - G\right) = -\frac{2\, i\, \alpha\, m \left(A_n\, e^{i k x_n} + B_n\, e^{-i k x_n}\right)}{k\, \hbar^2}$$

$$e^{-i k x_n} \left(-A_n\, e^{2 i k x_n} + B_n + F\, e^{2 i k x_n} - G\right) = -2\, i\, \beta \left(A_n\, e^{i k x_n} + B_n\, e^{-i k x_n}\right)$$

Now we have these two equations that we will solve for A_{n+1} and B_{n+1} then construct the matrix M_n which is the coefficient matrix of A_n and B_n and represent the Transfer Matrix

$$A_{n+1}\, e^{i k x_n} + B_{n+1}\, e^{-i k x_n} = A_n\, e^{i k x_n} + B_n\, e^{-i k x_n}$$

$$e^{-i k x_n} \left(-A_n\, e^{2 i k x_n} + B_n + F\, e^{2 i k x_n} - G\right) = -2\, i\, \beta\, (A_n + B_n)$$

$$\left\{A_{n+1} = -i \left(\beta\, A_n + i\, A_n + \beta\, B_n\, e^{-2 i k x_n}\right),\ B_{n+1} = i\, \beta\, A_n\, e^{2 i k x_n} + i\, \beta\, B_n + B_n\right\}$$

$$M_n = \begin{pmatrix} 1 - i\, \beta & -i\, \beta\, e^{-2 i k x_n} \\ i\, \beta\, e^{2 i k x_n} & 1 + i\, \beta \end{pmatrix}$$

Arranging the matrices yields

$$\begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} = M_n \begin{pmatrix} A_n \\ B_n \end{pmatrix}$$

$$\begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 1 - i\, \beta & -i\, e^{-2 i k x_n}\, \beta \\ i\, e^{2 i k x_n}\, \beta & i\, \beta + 1 \end{pmatrix} \left| \begin{pmatrix} A_n \\ B_n \end{pmatrix} \right.$$

Let's Assume That we have three barriers, so n = 0, 1 , 2. We will now observe the behavior of the relationships between coefficients

$$\begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} = M_n \begin{pmatrix} A_n \\ B_n \end{pmatrix}$$

$$\begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} = \begin{pmatrix} 1 - i\, \beta & -i\, \beta\, e^{-2 i k x_n} \\ i\, \beta\, e^{2 i k x_n} & 1 + i\, \beta \end{pmatrix} \left| \begin{pmatrix} A_n \\ B_n \end{pmatrix} \right.$$

Setting n = 2, 1 ,0. Respectively, we get:

$$\begin{pmatrix} A_3 \\ B_3 \end{pmatrix} = M_2 \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$$

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = M_1 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = M_0 \begin{pmatrix} A_0 \\ B_0 \end{pmatrix}$$

Combining them in one equation yields:

$$\begin{pmatrix} A_3 \\ B_3 \end{pmatrix} = M_0\, M_1\, M_2 \begin{pmatrix} A_0 \\ B_0 \end{pmatrix}$$

We can now generalize this for n = 0, 1 , 2 ..., N:

$$\begin{pmatrix} A_{N+1} \\ B_{N+1} \end{pmatrix} = \prod_{n=0}^{N} M_n \begin{pmatrix} A_0 \\ B_0 \end{pmatrix}$$

Here you can Play with N to see the resultant matrix:

N

$$\begin{pmatrix} A_6 \\ B_6 \end{pmatrix} = M_0\, M_1\, M_2\, M_3\, M_4\, M_5 \begin{pmatrix} A_0 \\ B_0 \end{pmatrix}$$

Now let's check the determinant of M_n and check if it yields unity

$$M_n = \begin{pmatrix} 1 - i\, \beta & -i\, \beta\, e^{-2 i k x_n} \\ i\, \beta\, e^{2 i k x_n} & 1 + i\, \beta \end{pmatrix}$$

$$\mathrm{M} = \prod_{n=0}^{N} M_n$$

$$|M| = 1$$

From previous calculations of the B.C. we obtain these relations between the Coefficients:

$$A_{n+1} = -i \left(\beta\, A_n + i\, A_n + \beta\, B_n\, e^{-2 i k x_n}\right)$$

$$B_{n+1} = i\, \beta\, A_n\, e^{2 i k x_n} + i\, \beta\, B_n + B_n$$

Taking two wave functions, one represents far right incedent wave with A_0 and B_0 as its coefficients, and one represents far left Transmitted wave with coefficients A_{N+1} and B_{N+1}, Then setting $B_{N+1} = 0$, and Solving for T = $\frac{A_{N+1}}{A_0}$

$$A_{N+1} = -i \left(A_0\, \beta + i\, A_0 + \beta\, B_0\, e^{-2 i k x_0}\right)$$

$$B_{N+1} = i\, A_0\, \beta\, e^{2 i k x_0} + i\, \beta\, B_0 + B_0$$

$$A_0\, \beta\, e^{2 i k x_0} + (\beta - i)\, B_0 = 0$$

$$A_0 \to \frac{i\, B_0\, e^{-2 i k x_0} - \beta\, B_0\, e^{-2 i k x_0}}{\beta}$$

$$T = \left|\frac{A_{N+1}}{A_0}\right|^2 = \frac{1}{\beta^2 + 1}$$

Now Recall M and lets check if T = $\frac{1}{|M_{22}|^2}$

$$M_n = \begin{pmatrix} 1 - i\, \beta & -i\, \beta\, e^{-2 i k x_n} \\ i\, \beta\, e^{2 i k x_n} & 1 + i\, \beta \end{pmatrix}$$

$$\mathrm{M} = \prod_{n=0}^{N} M_n$$

$$M_{22} = 1 + i\, \beta$$

$$\frac{1}{|M_{22}|^2} \stackrel{?}{=} T$$

$$\frac{1}{|M_{22}|^2} = \frac{1}{\beta^2 + 1} = T$$

Bonus: Checking whether T + R = 1 holds

$$A_{N+1} = -i \left(A_0\, \beta + i\, A_0 + \beta\, B_0\, e^{-2 i k x_0}\right)$$

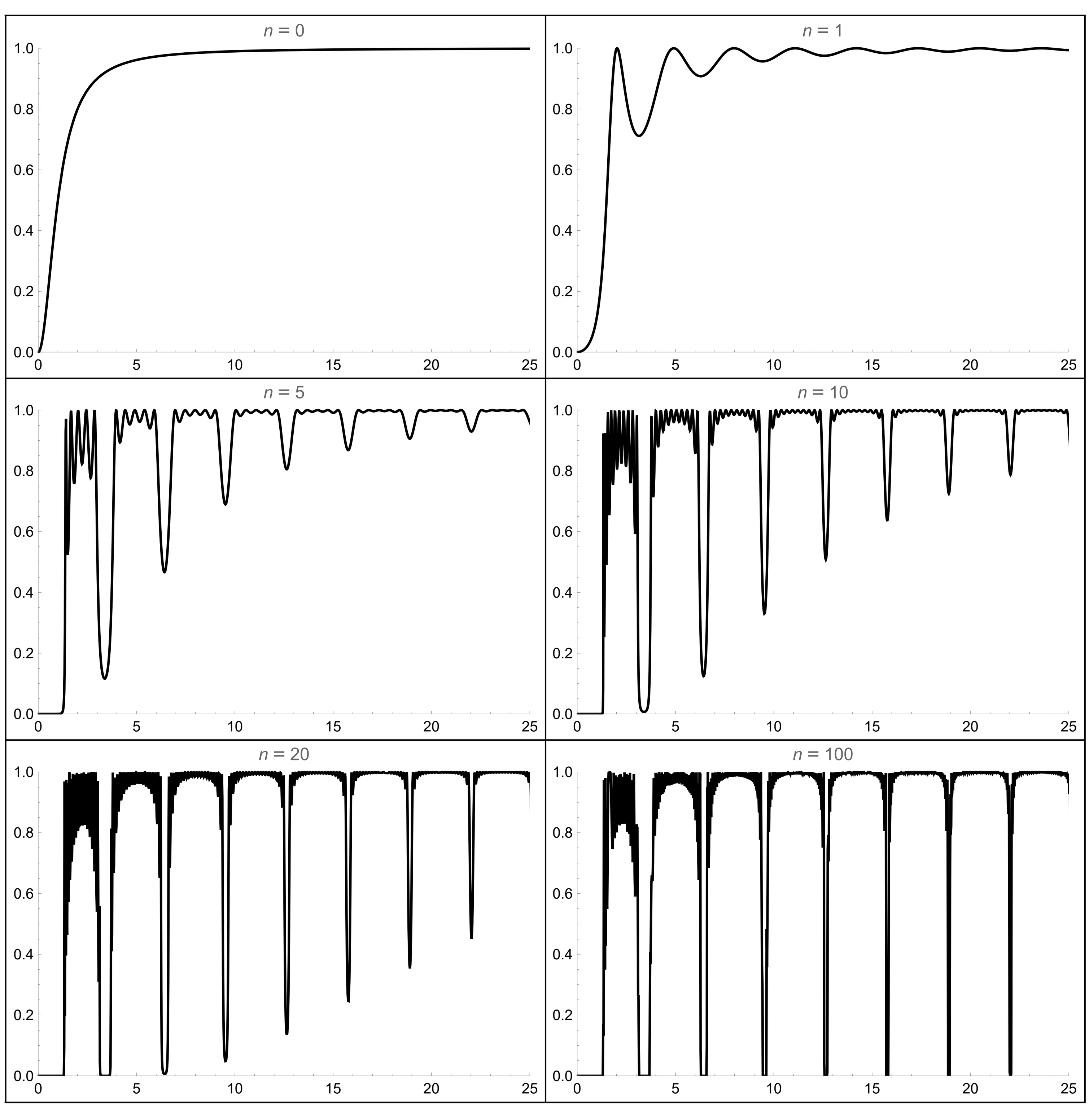
$$B_{N+1} = i\, A_0\, \beta\, e^{2 i k x_0} + i\, \beta\, B_0 + B_0$$

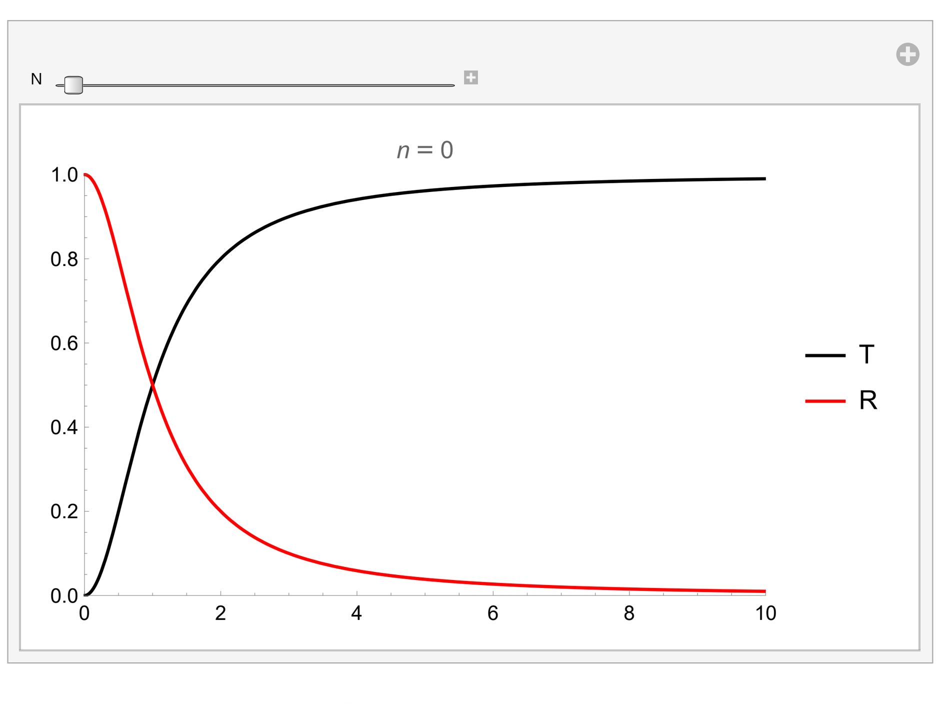
$$A_0\, \beta\, e^{2 i k x_0} + (\beta - i)\, B_0 = 0$$

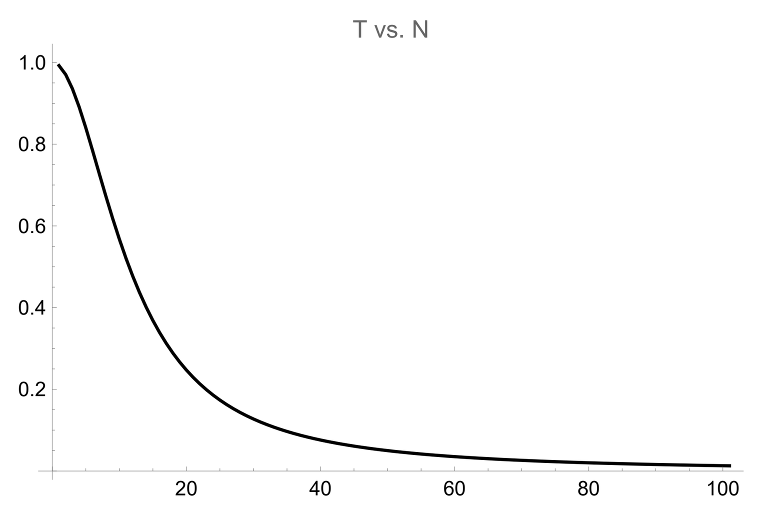
$$B_0 \to -\frac{A_0\, \beta\, e^{2 i k x_0}}{\beta - i}$$

$$R = \frac{\beta^2}{\beta^2 + 1}$$

$$R + T = \frac{\beta^2}{\beta^2 + 1} + \frac{1}{\beta^2 + 1} = 1$$





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