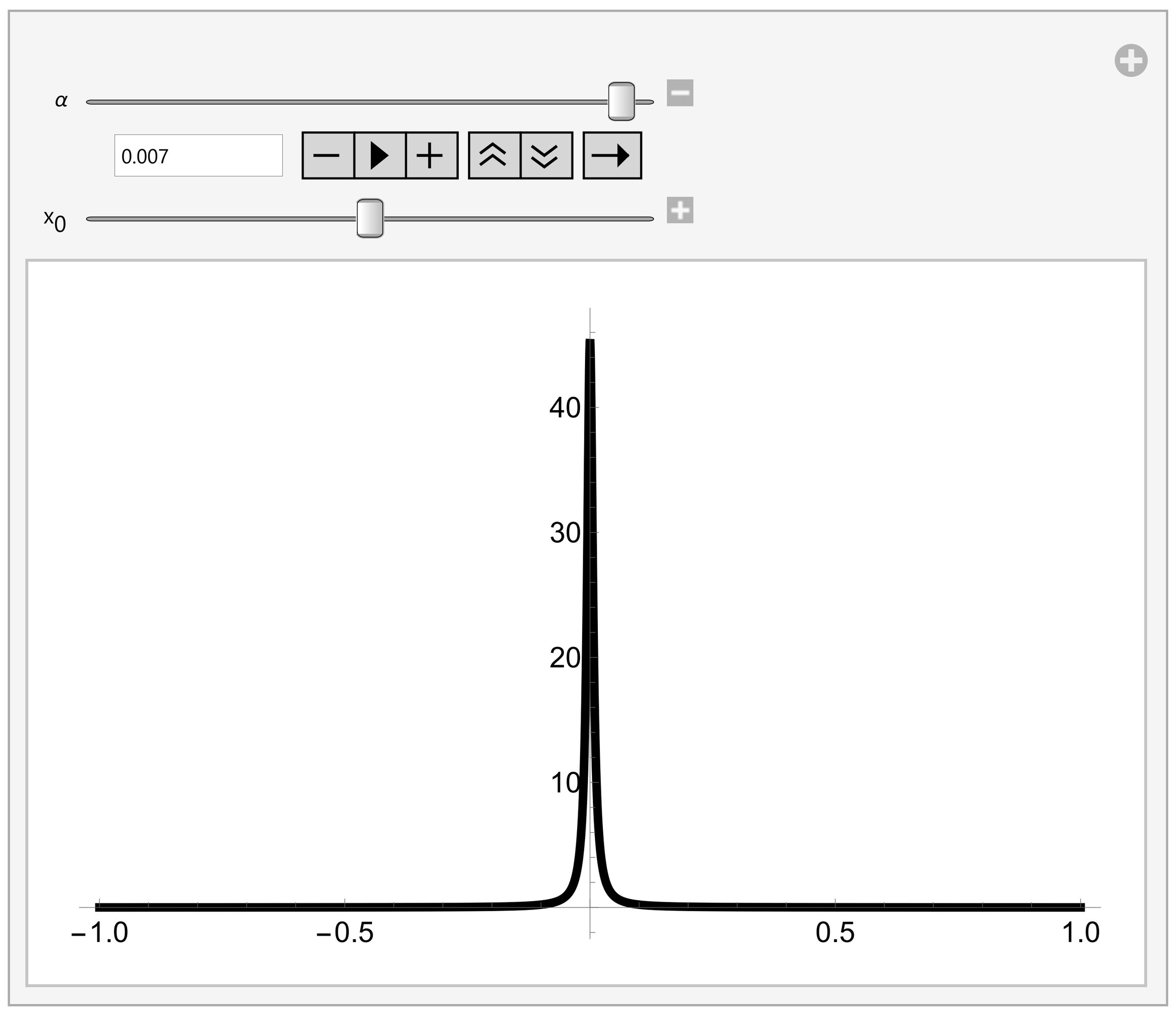
Here are some Dirac Delta function models, we can see that they peak at x = x_0 and will converge to zero everywhere else when applying the proper limit.

$$f(x) = \lim_{\alpha \to 0} \frac{\alpha}{\pi\left(\alpha^2 + x^2\right)}$$

In[∘]:=
```
Manipulate[Plot[α/(π * ((x - x0)^2 + α^2)), {x, -1, 1}, PlotRange → All, PlotStyle → {AbsoluteThickness[3.], Black}], {α, .5, 0.000001}, {{x0, 0}, -1, 1}]
```

Out[∘]=



$$\Theta(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

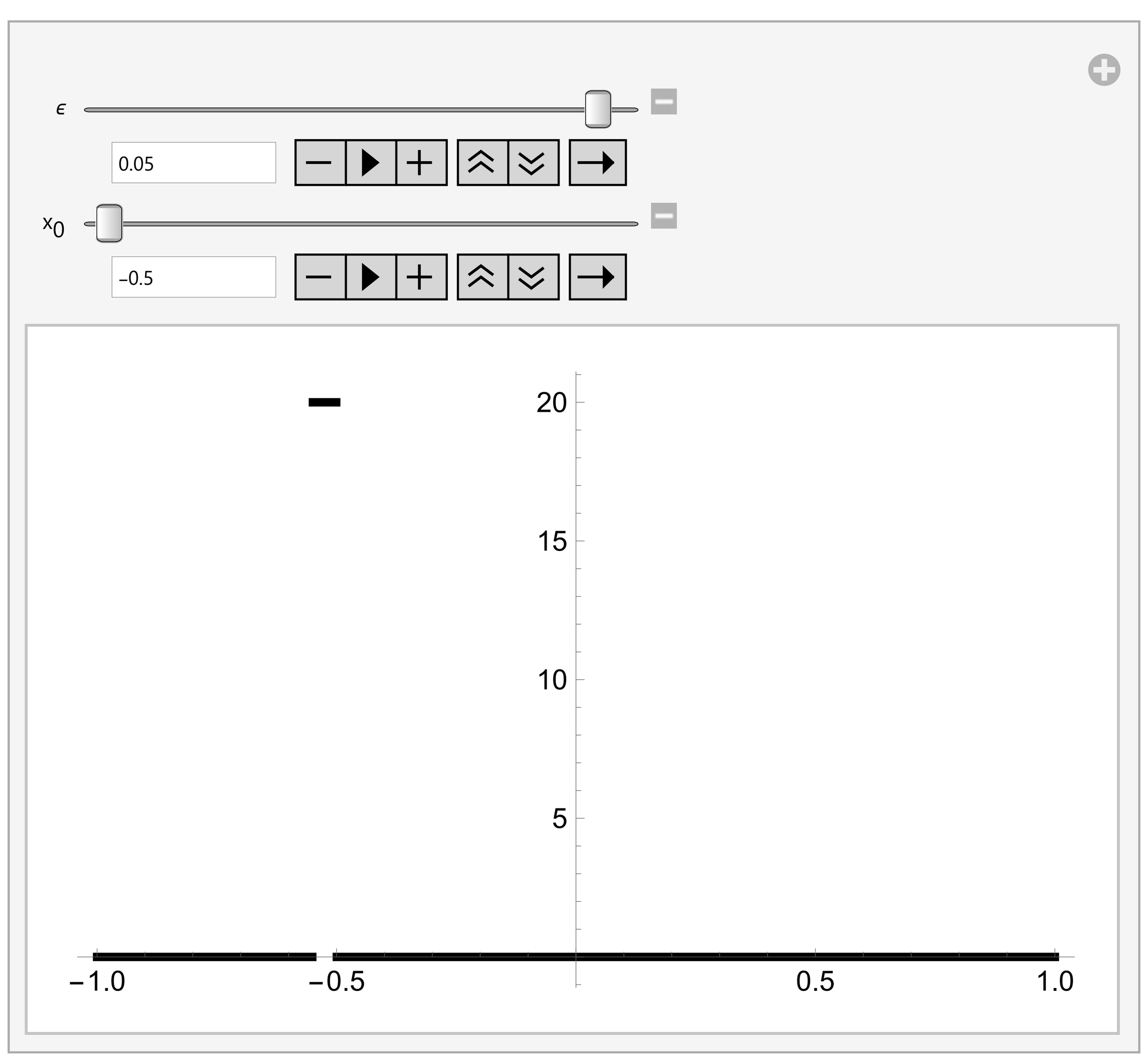
$$\rho(x) = \lim_{\epsilon \to 0} \frac{\Theta(x) + \Theta(x + \epsilon)}{\epsilon}$$

In[∘]:=
```
Θ[x_] := UnitStep[x];
```

In[∘]:=
```
ρ[x_, ϵ_] := (Θ[x + ϵ] - Θ[x])/ϵ;
```

In[∘]:=
```
Manipulate[Plot[ρ[x - x0, ϵ], {x, -1, 1}, PlotStyle → {AbsoluteThickness[3.], Black}], {ϵ, 1, 0.02}, {{x0, 0}, -.5, 1}]
```

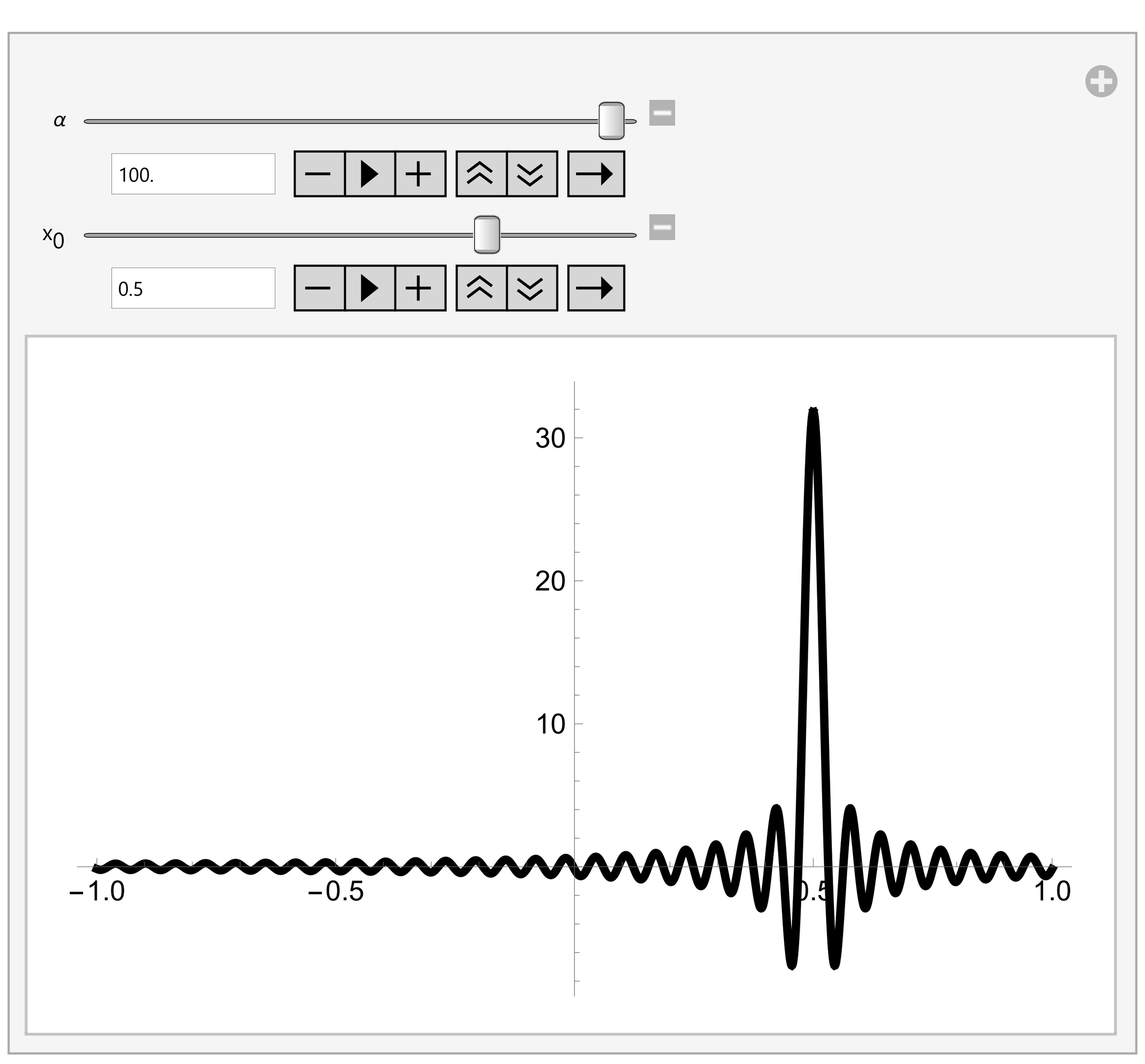
Out[∘]=



$$f(x) = \lim_{\alpha \to \infty} \frac{\sin(\alpha x)}{\pi x}$$

In[∘]:=
```
Manipulate[Plot[Sin[α * (x - x0)]/(π * (x - x0)), {x, -1, 1}, PlotStyle → {AbsoluteThickness[3.], Black}, PlotRange → All], {α, 1, 100}, {{x0, 0}, -1, 1}]
```

In[∘]:=



Ibraheem Al-Yousef, PHYS310.