Q.1 $g(t)=\begin{cases}-1 & ,-\pi\le t\le 0\\ 1 & ,\ 0\le t\le\pi\end{cases}$,

We can observe that

$$g(-t)=\begin{cases}-1 & ,-\pi\le -t\le 0\\ 1 & ,\ 0\le -t\le\pi\end{cases}=\begin{cases}-1 & ,\ \pi\ge t\ge 0\\ 1 & ,\ 0\ge t\ge -\pi\end{cases}=-g(t)$$

Which means this function is an odd function, and hence we will only expand it using sin terms of Fourier expansion.

$$Forier\ expansion\ of\ the\ function\ above\ is\ g(t)=\sum_{i=1}^{n} b_n\times\sin nt$$

$$where\ b_n=\frac{1}{\pi}*\int_{-\pi}^{\pi} g(t)\times\sin nt=\frac{1}{\pi}\left[\int_{-\pi}^{0}-\sin nt+\int_{0}^{\pi}\sin nt\right]$$

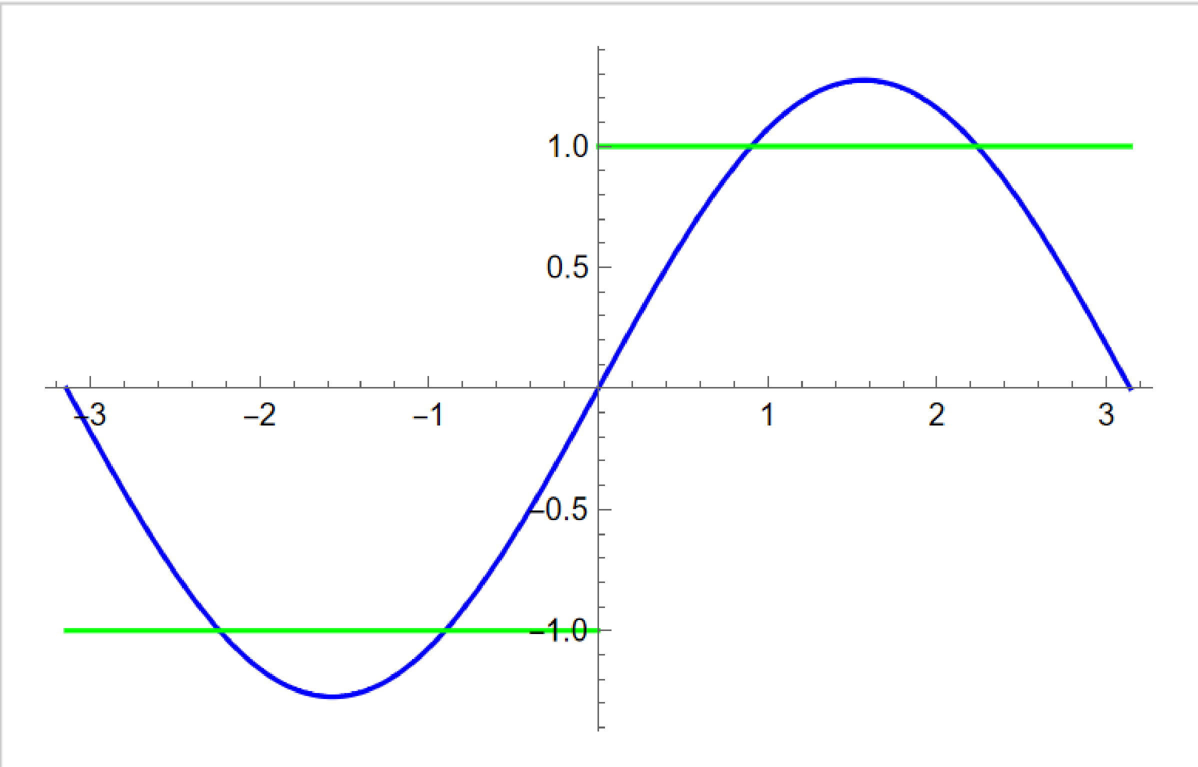
$$=\frac{1}{\pi}\left[\frac{\cos nt}{n}\bigg|_{-\pi}^{0}-\frac{\cos nt}{n}\bigg|_{0}^{\pi}\right]=\frac{1}{n\pi}[\cos 0-\cos n\pi-\cos n\pi+\cos 0]$$

$$taking\ \cos n\pi=(-1)^n, b_n=\frac{1}{n\pi}[2-2(-1)^n]$$

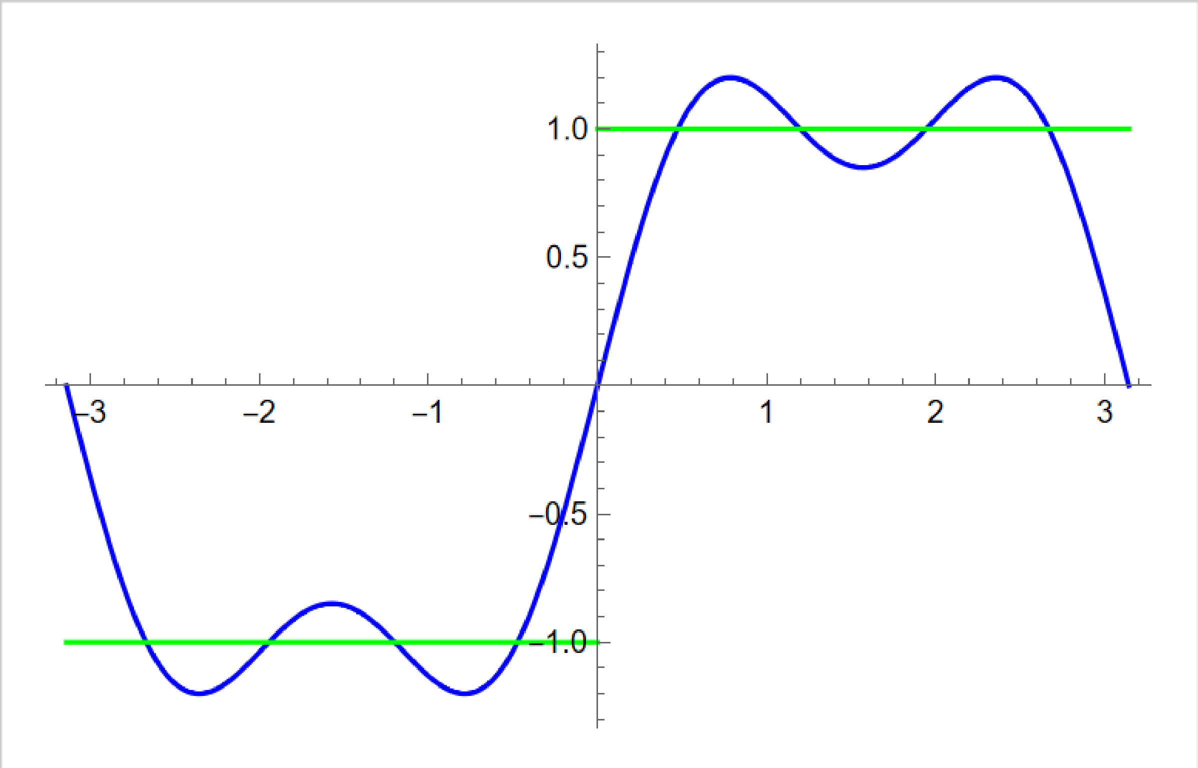
$$b_n=\frac{2-2(-1)^n}{n\pi}$$

$$g(t)=\sum_{i=1}^{n}\frac{2-2(-1)^n}{n\pi}\times\sin nt$$

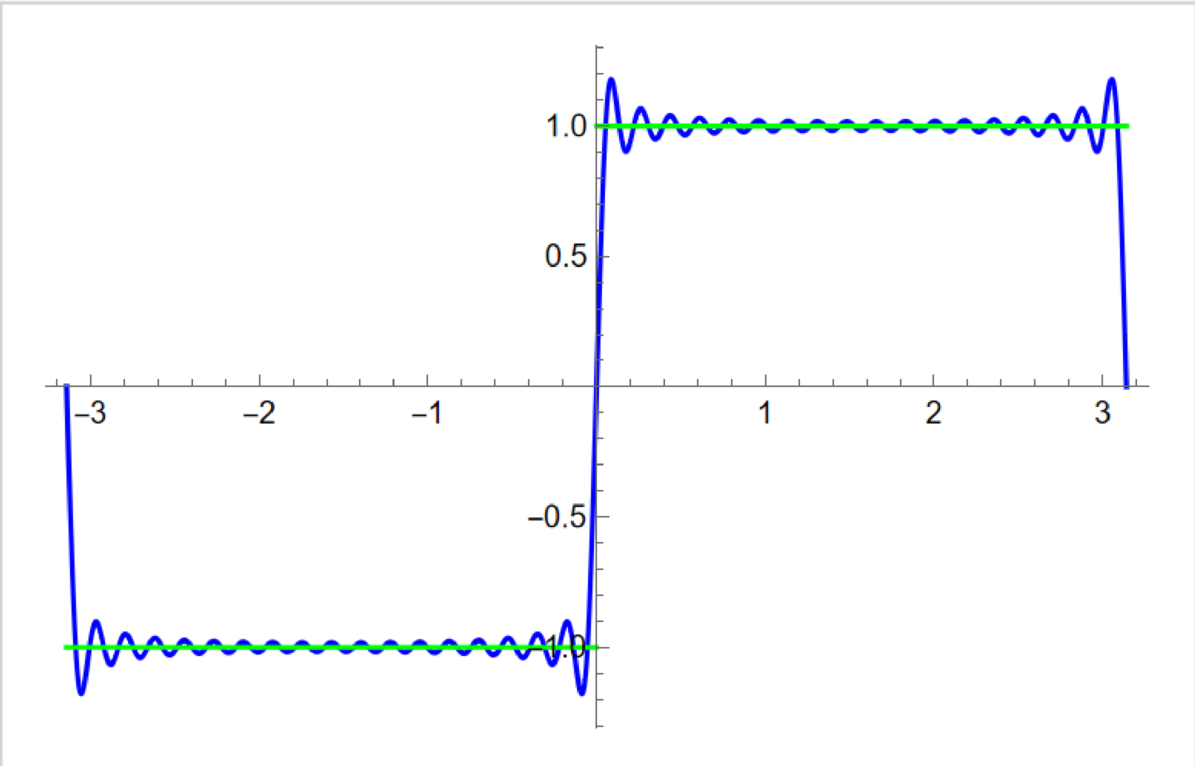
Plotting the first two terms of Fourier expansion alongside $g(\mathrm{t})$:



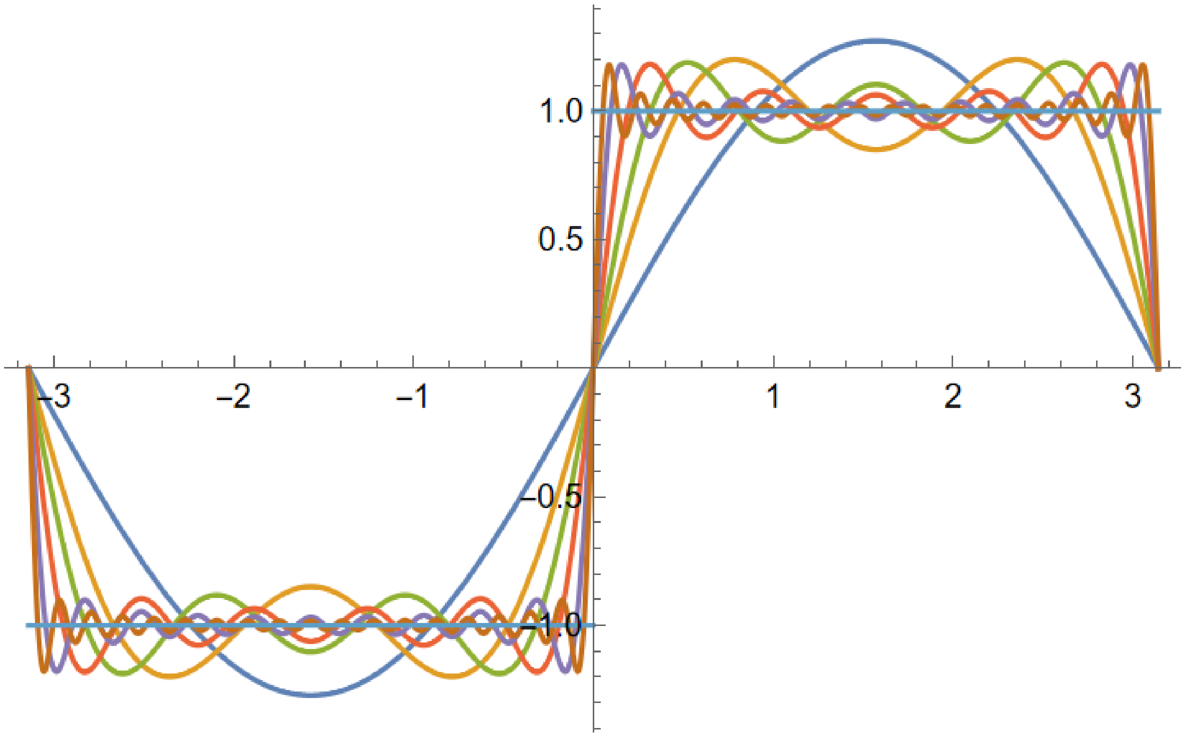
Plotting the first three terms of Fourier expansion alongside $g(\mathrm{t})$:



Plotting the first 35 term of Fourier expansion alongside $g(\mathrm{t})$, where I believe maximum convergence is achieved:



A combined plot for multiple terms of Fourier expansion, up to 35 showing how more terms of Fourier expansion yield more accurate approximation:



The fluctuation at the boundaries:

We can notice that the Fourier series expansion has a disturbance around its boundaries and also around any jump discontinuity. This is phenomenon known as Gibbs Phenomenon which is a ringing effect caused by the limitation of the *finite* sum of Fourier series terms. However, taking the *infinite* sum of Fourier series terms will generate a smooth convergent curve that exactly matches the function before the expansion. A detailed article is attached as well as numerical code.