Numerical Homework 2

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(*Defining descretizing constants*)

Nx = 100; xmin = -10.; xmax = 10.; dx = (xmax - xmin)/(Nx + 1); Nt = 500; dt := (2 π/Ω)/Nt;

(*Exact wavefunction for harmonic oscillator*)

psi[n_, x_] := 1/Sqrt[Sqrt[π]] 1/Sqrt[2^n n!] HermiteH[n, x] Exp[-1/2 x^2] // N;

(*Numerical operators: 1D second derivative + position operator*)

d2 = 1/dx^2 SparseArray[{{i_, i_} → -2., {i_, j_} /; Abs[i - j] == 1 → 1.}, {Nx, Nx}]; X = SparseArray[{{i_, i_} → xmin + dx i, {Nx, Nx}}];

(*General form of 1D Hamiltonian*)

H[t_] := -1/2 d2 + V[t]

(*Cayley’s form of time-evolution operator*)

Uplus[t_] := IdentityMatrix[Nx] + 1/2 I H[t + dt/2] dt // N;

Uminus[t_] := IdentityMatrix[Nx] - 1/2 I H[t + dt/2] dt // N;

(*Potential function defintion, with a driving frequency*)

V[t_] := 1/2 X.X - X * f[t];

(* Simulation protocol*)

Simulate[ω_] := (Ω = ω;

  f[t_] := Sin[Ω t];
  xc[t_] = Simplify[Integrate[f[tp] Sin[t - tp], {tp, 0, t}]];
  Ψ[n_, x_, t_] = Simplify[psi[n, x - xc[t]] Exp[I (-(n + 0.5) t + xc'[t] (x - xc[t]/2) + 1/2 Integrate[f[tp] × xc[tp], {tp, 0, t}])]] // N;

  Psi = Eigenvectors[H[0]][[Nx]]/Sqrt[dx] // N;

  data = Table[Psi = LinearSolve[Uplus[(i - 1) dt], Uminus[(i - 1) dt].Psi];

    If[Mod[i, Nt/100] == 0, exactψ[x_] = Ψ[0, x, i dt];
     exactψ0[x_] = psi[0, x - f[i dt]];
     Show[{Plot[{Re[exactψ[x]], Im[exactψ[x]], Abs[exactψ[x]], Abs[exactψ0[x]]}, {x, -5, 5}, PlotRange → {-1, 1}, PlotStyle → {Red, Blue, Black, {Dashed, Green}}, AxesLabel → {"x", "|Ψ|"},
        PlotLegends → {(TraditionalForm@Re[ψ] // ToString) <> "_Exact", (TraditionalForm@Im[ψ] // ToString) <> "_Exact", (TraditionalForm@Abs[ψ] // ToString) <> "_Exact",
          (TraditionalForm@Abs[ψ0] // ToString) <> "_Exact"}], ListPlot[{Re[Psi], Im[Psi], Abs[Psi]}, PlotRange → {-1, 1},
        PlotLegends → {(TraditionalForm@Re[ψ] // ToString) <> "_Numerical", (TraditionalForm@Im[ψ] // ToString) <> "_Numerical", (TraditionalForm@Abs[ψ] // ToString) <> "_Numerical"},
        DataRange → {xmin + dx, xmax - dx}, PlotStyle → {Red, Blue, Black}]}], Nothing], {i, 1, Nt}];

  ListAnimate[data, 20, AnimationRunning → False])
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11.38

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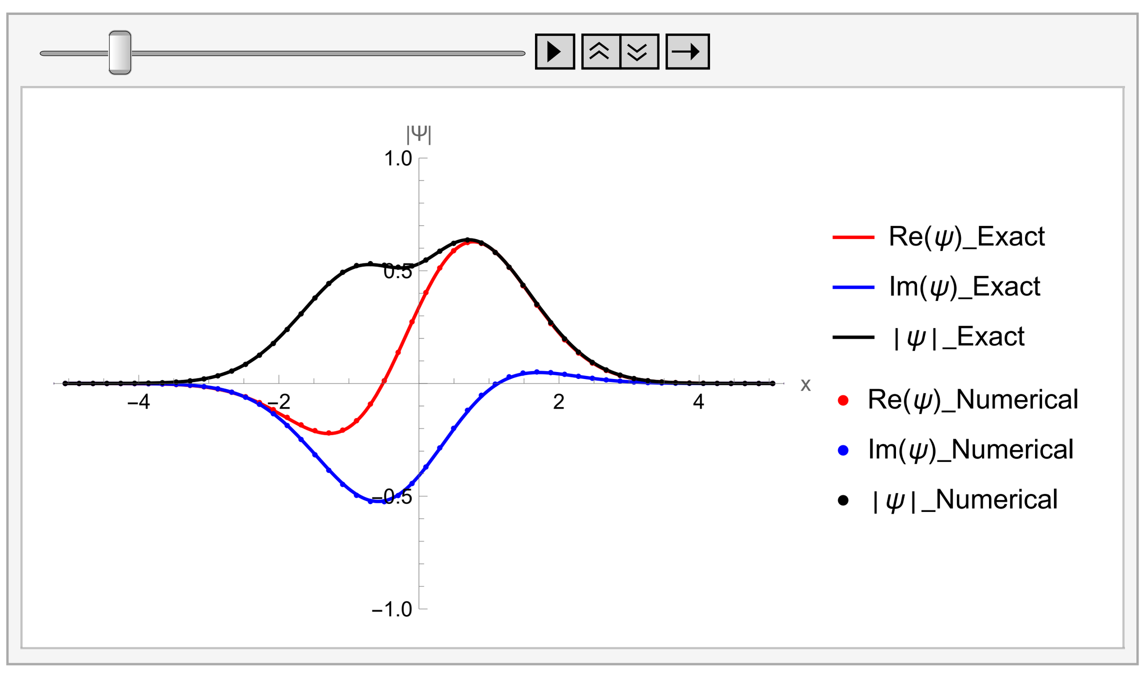
f[t_] := 0 * Sin[Ω t];
Ω = 0.5;
Psi = (Eigenvectors[H[t]][[Nx]]/Sqrt[dx] + Eigenvectors[H[t]][[Nx-1]]/Sqrt[dx])/Sqrt[2];

Data = Table[Psi = LinearSolve[Uplus[(i - 1) dt], Uminus[(i - 1) dt].Psi];

  If[Mod[i, Nt/100] == 0, exactψ[x_] = (psi[0, x] Exp[-I i dt/2] - psi[1, x] Exp[-3 I i dt/2])/Sqrt[2];

   Show[{Plot[{Re[exactψ[x]], Im[exactψ[x]], Abs[exactψ[x]]}, {x, -5, 5}, PlotRange → {-1, 1}, PlotStyle → {Red, Blue, Black}, AxesLabel → {"x", "|Ψ|"},
      PlotLegends → {(TraditionalForm@Re[ψ] // ToString) <> "_Exact", (TraditionalForm@Im[ψ] // ToString) <> "_Exact", (TraditionalForm@Abs[ψ] // ToString) <> "_Exact"}],
     ListPlot[{Re[Psi], Im[Psi], Abs[Psi]}, PlotRange → {-1, 1},
      PlotLegends → {(TraditionalForm@Re[ψ] // ToString) <> "_Numerical", (TraditionalForm@Im[ψ] // ToString) <> "_Numerical", (TraditionalForm@Abs[ψ] // ToString) <> "_Numerical"},
      DataRange → {xmin + dx, xmax - dx}, PlotStyle → {Red, Blue, Black}]}], Nothing], {i, 1, Nt}];
ListAnimate[Data, 20, AnimationRunning → False]
```



11.39 a): $\Omega = \frac{1}{5}\,\omega$

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Simulate[0.2]
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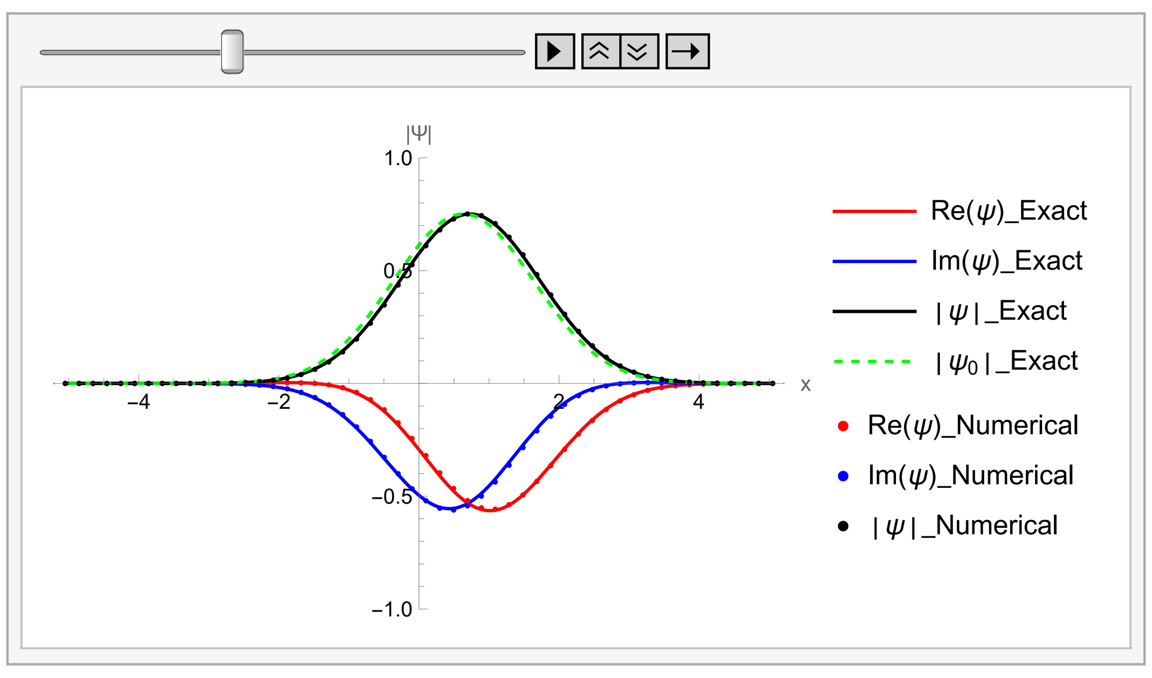

11.39 a): $\Omega = 5\,\omega$

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Simulate[5.]
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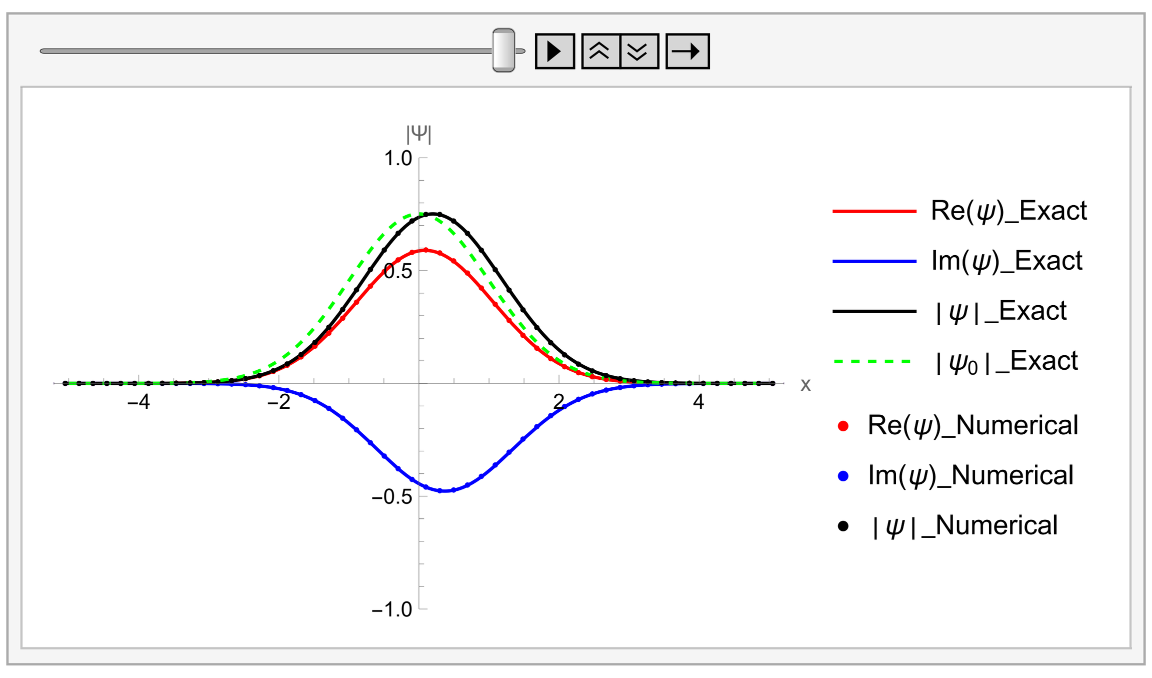

11.39 a): $\Omega = \frac{6}{5}\,\omega$

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Simulate[6./5.]
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