$$V_{\text{eff}} = \frac{l(l+1)\hbar^2}{2m_e R^2} - \frac{e^2 k}{R}$$

Using the electronic units. i.e, $\hbar = 1, k = 1, m_e = 1, e = 1$:

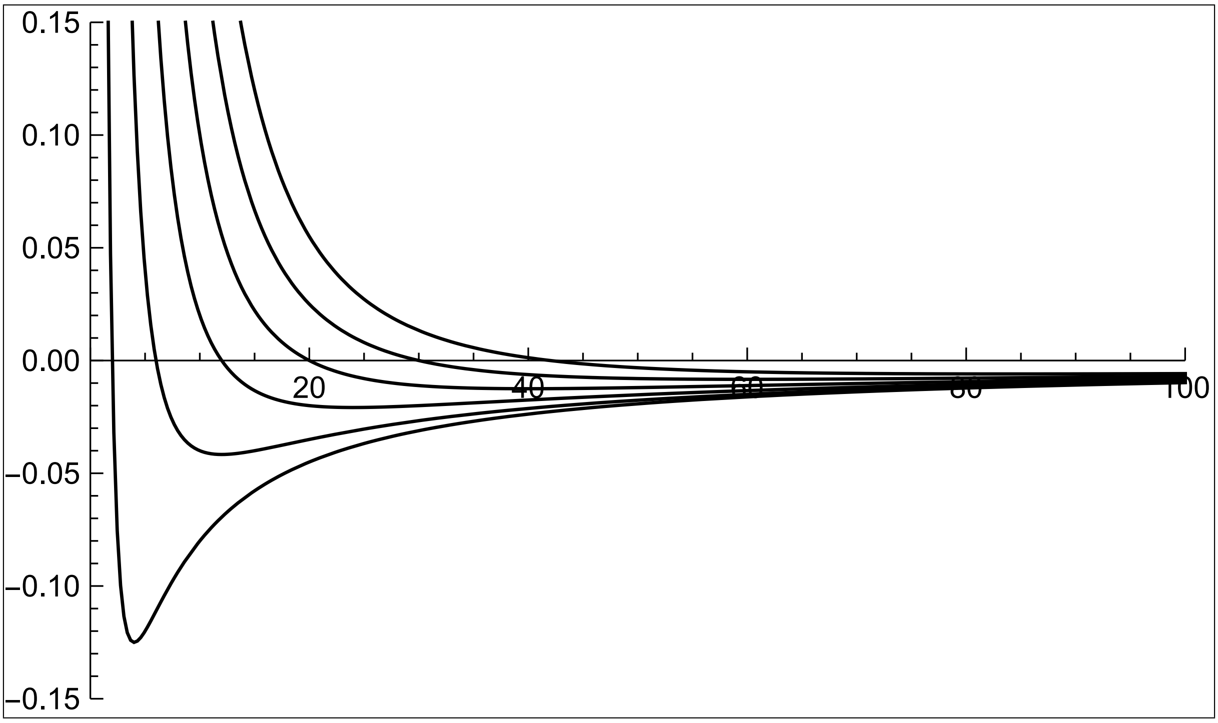


**Figure 1.** $V_{eff}(R)$ vs $R$ for $l$ = 1, 2, 3, 4, 5, 6

Now let's check the derivative of $V_{eff}(R)$ to get the minimums of the potential. I will call it $V_{eff}(R_*)$ Then plot it over the previous plot

$$\frac{\mathrm{dV}_{eff}}{\mathrm{dR}} = 0 \to \frac{e^2 k}{R^2} - \frac{l(l+1)\hbar^2}{m_e R^3} = 0 \to R_* = \frac{(l^2+l)\,\hbar^2}{e^2 k m_e} \to V_{eff}(R_*) = -\frac{(e^2 k m_e)^2}{2m_e \hbar^2 l(l+1)} = -\frac{e^4 k^2 m_e}{2l(l+1)\hbar^2}$$

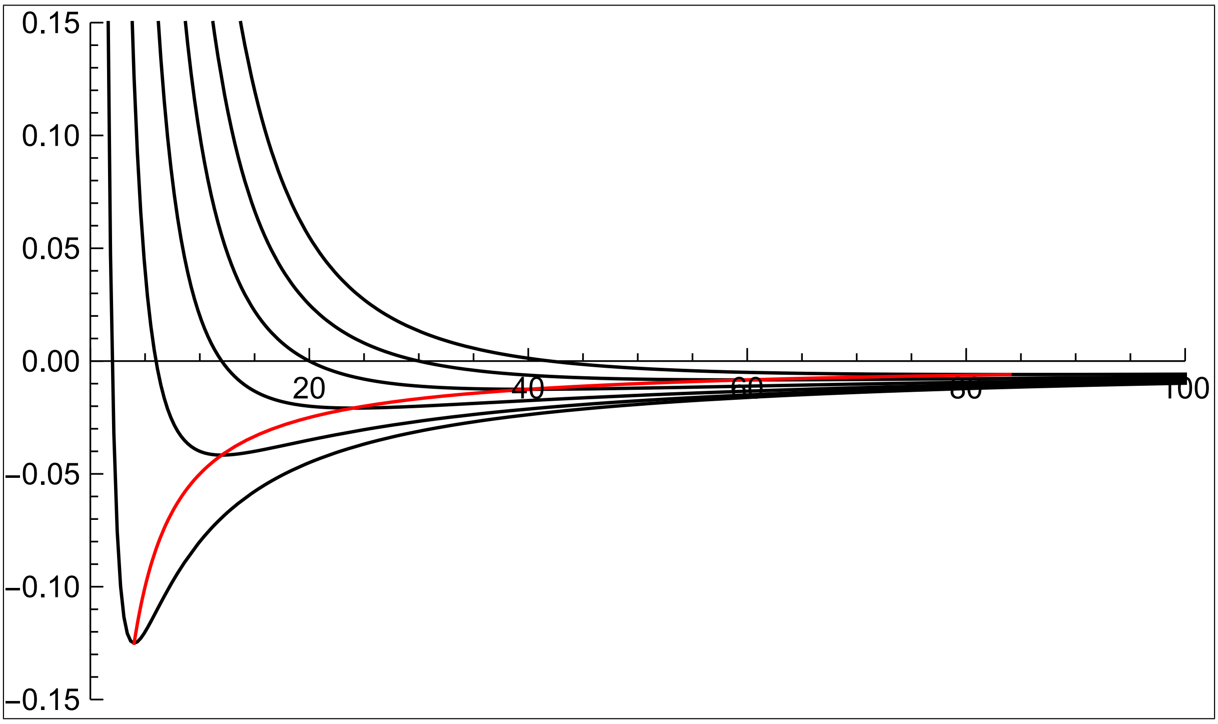


**Figure 2.** $V_{eff}(R_*)$ vs $R_*$ on the previous plot for $l$ = 1, 2, 3, 4, 5, 6

Now let's check what happens to the maximum orbital quantum number $l$ at $n >> 1$. We will replace $l$ by $n-1$ in $V_{eff}(R_*)$ to see what we will get:

$$V_{eff}(R_*) = -\frac{(e^2km_e)^2}{2m_e\hbar^2 l(l+1)} = -\frac{(e^2km_e)^2}{2m_e\hbar^2 n(n-1)},\ \text{Since } n >> 1, V_{eff}(R_*) \approx -\frac{(e^2km_e)^2}{2m_e\hbar^2 n^2}$$

$$\text{Which looks quite familiar, remembering that } E_n = -\frac{(e^2km_e)^2}{2m_e\hbar^2 n^2}$$

$$\therefore V_{eff}(R_*) \approx E_n;\ \text{For } l_{max} \text{ and } n >> 1$$

Now I will plot $V_{eff}(R_*)$ and $E_n$ vs $n$ to show that at the high values of the principle quantum number $n$, $V_{eff}(R_*) \approx E_n$

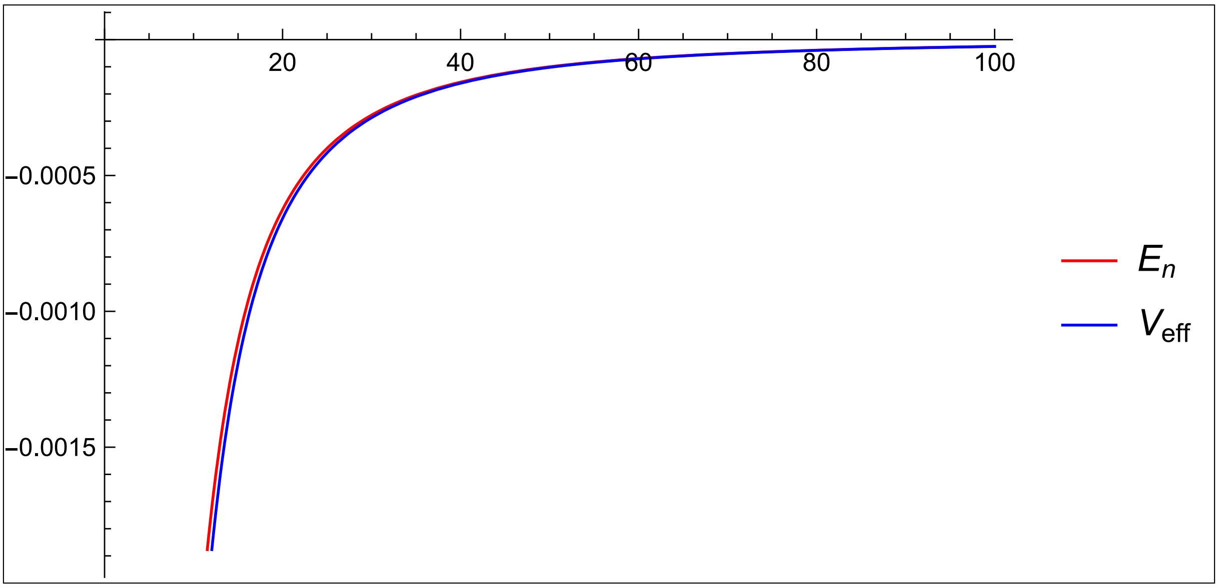


**Figure 3.** $V_{eff}(R_*)$ and $E_n$ vs $n$.