Ibraheem Al-Yousef

PHYS422 HW.1

28/1/2023

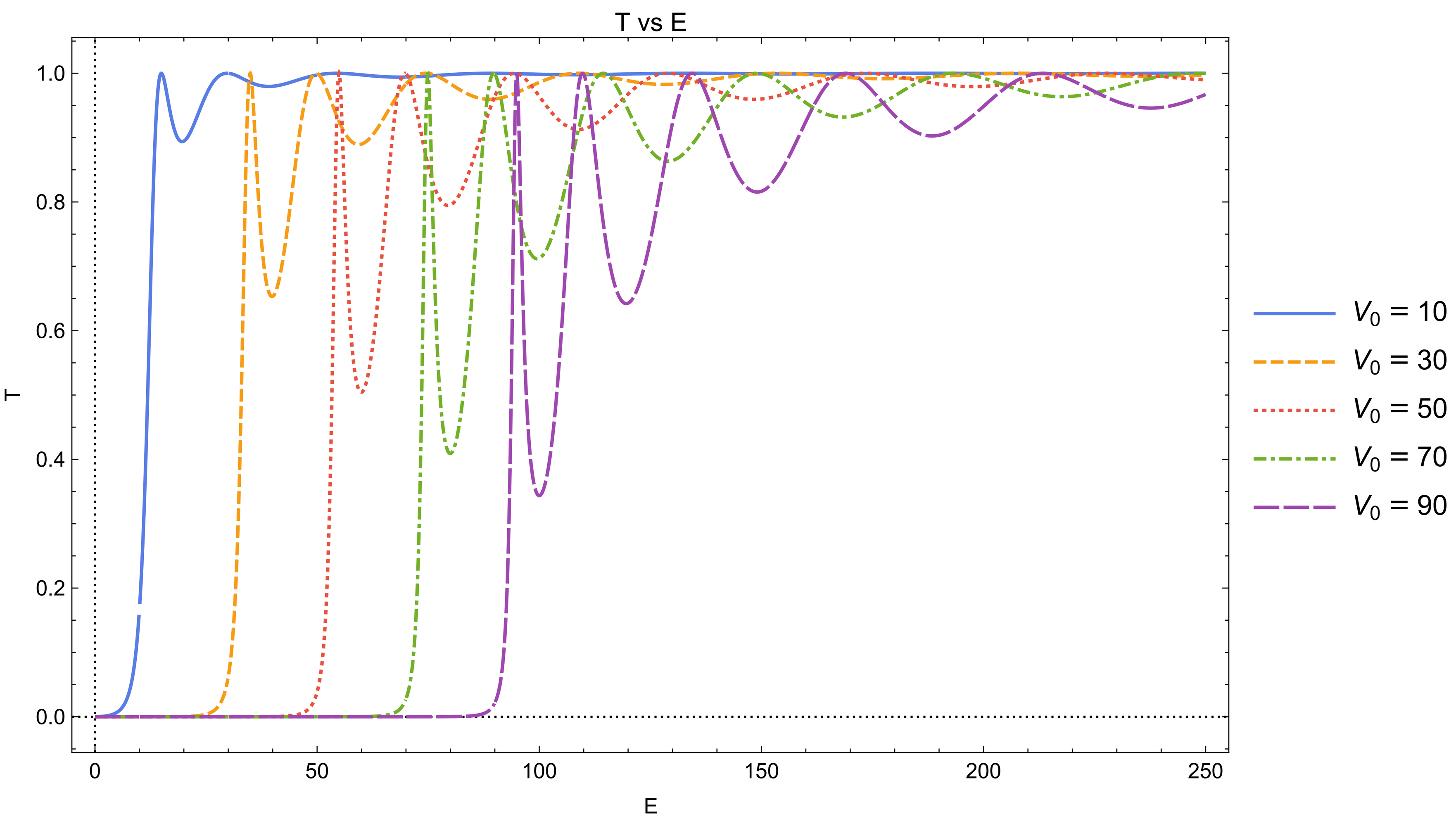
Problem 1):

```
ψ1 = A e^(i k1 x) + b e^(-i k1 x);
ψ2 = d e^(-i k2 x) + c e^(i k2 x);
ψ3 = f e^(i k3 x);
fSol = f /. Solve[{ψ1 == ψ2 /. x → 0, D[ψ1, x] == D[ψ2, x] /. x → 0, ψ3 == ψ2 /. x → a, D[ψ3, x] == D[ψ2, x] /. x → a}, {b, c, d, f}][[1]];
```

```
T = 1/A^2 FullSimplify@(ComplexExpand@Abs[fSol] /. k1 → k3 /. k2 → Sqrt[2 m (E - V0)/ħ^2] /. k3 → Sqrt[2 m E/ħ^2])^2 /. a → 1 /. ħ → 1 /. m → 1
```

$$\frac{4\,E\,(E - V_0)}{4\,E^2 - 4\,E\,V_0 + \text{Sin}\left[\sqrt{2}\,\sqrt{E - V_0}\right]^2 V_0^2}$$

```
plt = Table[T, {V0, 10, 100, 20}];
Plot[plt, {E, 0, 250}, PlotRange → All, PlotTheme → {"Scientific", "BoldColor", "Monochrome"}, FrameLabel → {"E", "T"},
 PlotLegends → Table[TraditionalForm[V0 == t], {t, 10, 100, 20}], ImageSize → Large, PlotLabel → "T vs E"]
```



We can notice that the Transmission coefficient is almost zero when E < V_0. Moreover, when E > V_0, the transmission is not always 1, it oscillates closer to 1 as E gets bigger.

Problem 3):

We first solve Schrodinger's equation for two regions, the first is when 0<x<a. The second is when x>a:

```
ψ1 = (ψ[x] /. Assuming[m > 0 && E > 0 && V0 > 0, DSolve[{-ħ^2/(2 m) D[ψ[x], {x, 2}] - V0 ψ[x] == E ψ[x], ψ[0] == 0}, ψ[x], x]])[[1]] /. C[1] → A;
Assuming[m > 0 && E > 0 && V0 > 0 && a > 0, FullSimplify[ψ1] /. Sqrt[2 (E + V0) m]/ħ → α] // TraditionalForm
```

$-2\,i\,A\,\sin(\alpha\,x)$

```
ψ2 = (ψ[x] /. DSolve[-ħ^2/(2 m) D[ψ[x], {x, 2}] == E ψ[x], ψ[x], x])[[1]];
Assuming[m > 0 && E > 0 && V0 > 0 && a > 0, FullSimplify[ψ2] /. Sqrt[2 E m]/ħ → k] // TraditionalForm
```

$c_1 \cos(k\,x) + c_2 \sin(k\,x)$

$\alpha = \frac{\sqrt{2\,(E+V_0)\,m}}{\hbar}$; $k = \frac{\sqrt{2\,E\,m}}{\hbar}$. Now we apply the continuity of the wavefunction and its derivative at the boundary x=a. The solutions of the coefficients are in terms of A:

```
Sol = Solve[{ψ2 == ψ1, D[ψ1, x] == D[ψ2, x]} /. x → a, {C[1], C[2]}];
Assuming[m > 0 && E > 0 && V0 > 0 && a > 0, FullSimplify[Sol] /. Sqrt[2 (E + V0) m]/ħ → α /. Sqrt[2 E m]/ħ → k][[1]] // TraditionalForm
```

$$\left\{c_1 \to -2\,i\,A\left(\sin(a\,\alpha)\cos(a\,k) - \sqrt{\frac{E+V_0}{E}}\cos(a\,\alpha)\sin(a\,k)\right),\ c_2 \to -2\,i\,A\left(\sin(a\,\alpha)\sin(a\,k) + \sqrt{\frac{E+V_0}{E}}\cos(a\,\alpha)\cos(a\,k)\right)\right\}$$

Problem 7):

a)

$$\langle x\rangle = \sqrt{\frac{h}{4\,\pi\,m\,\omega}}\ \langle 0 \mid a + a^{\dagger} \mid 0\rangle = 0$$

$$\langle x^2\rangle = \frac{h}{4\,\pi\,m\,\omega}\ \langle 0 \mid (a + a^{\dagger}) * (a + a^{\dagger}) \mid 0\rangle = \frac{h}{4\,\pi\,m\,\omega}\ \langle 0 \mid a\,a + a^{\dagger}\,a^{\dagger} + a\,a^{\dagger} + a^{\dagger}\,a \mid 0\rangle = \frac{h}{4\,\pi\,m\,\omega}\ \langle 0 \mid a\,a^{\dagger} \mid 0\rangle = \frac{h}{4\,\pi\,m\,\omega}$$

b)

$$\Delta x = \left[\langle x^2\rangle - \langle x\rangle^2\right]^{\frac{1}{2}} = \sqrt{\frac{h}{4\,\pi\,m\,\omega}}$$

c)

$$\langle p_x\rangle = i\sqrt{\frac{h\,m\,\omega}{4\,\pi}}\ \langle 0 \mid a - a^{\dagger} \mid 0\rangle = 0$$

$$\langle p_x^2\rangle = -\frac{h\,m\,\omega}{4\,\pi}\ \langle 0 \mid (a - a^{\dagger}) * (a - a^{\dagger}) \mid 0\rangle = -\frac{h\,m\,\omega}{4\,\pi}\ \langle 0 \mid a\,a + a^{\dagger}\,a^{\dagger} - a\,a^{\dagger} - a^{\dagger}\,a \mid 0\rangle = \frac{h\,m\,\omega}{4\,\pi}\ \langle 0 \mid a\,a^{\dagger} \mid 0\rangle = \frac{h\,m\,\omega}{4\,\pi}$$

d)

$$\Delta p_x = \left[\langle p_x^2\rangle - \langle p_x\rangle^2\right]^{\frac{1}{2}} = \sqrt{\frac{h\,m\,\omega}{4\,\pi}}$$

$$\Delta x\,\Delta p_x = \sqrt{\frac{h\,m\,\omega}{4\,\pi}}\ \sqrt{\frac{h}{4\,\pi\,m\,\omega}} = \frac{h}{4\,\pi}$$

This is a "minimum-uncertainty" wave packet because it has the minimum Δx possible and the minimum Δp_x possible. This restriction is described in the uncertainty principle $\Delta x\,\Delta p_x \geq \frac{h}{4\,\pi}$.

Problem 15):

a) "f" means l=3. Therefore $j = l \pm \frac{1}{2} \Longrightarrow j = \frac{5}{2}$ or $j = \frac{7}{2}$

b) for $j = \frac{5}{2} \Longrightarrow m_j = \pm\frac{5}{2},\ \pm\frac{3}{2},\ \pm\frac{1}{2}$

for $j = \frac{7}{2} \Longrightarrow m_j = \pm\frac{7}{2},\ \pm\frac{5}{2},\ \pm\frac{3}{2},\ \pm\frac{1}{2}$

c) The total number of m_j states $= 2\,j + 1$

For "f" state, for:

$j = \frac{7}{2}$ the total number of states is: $2 \times \frac{7}{2} + 1 = 8$

$j = \frac{5}{2}$ the total number of states is: $2 \times \frac{5}{2} + 1 = 6$

d) If we use m_l and m_s we will count the states for all possible values of j so the total number of states $= 2 \times (2\,l + 1)$

For "f" state, the total number of states is: $2 \times (3 \times 2 + 1) = 14$