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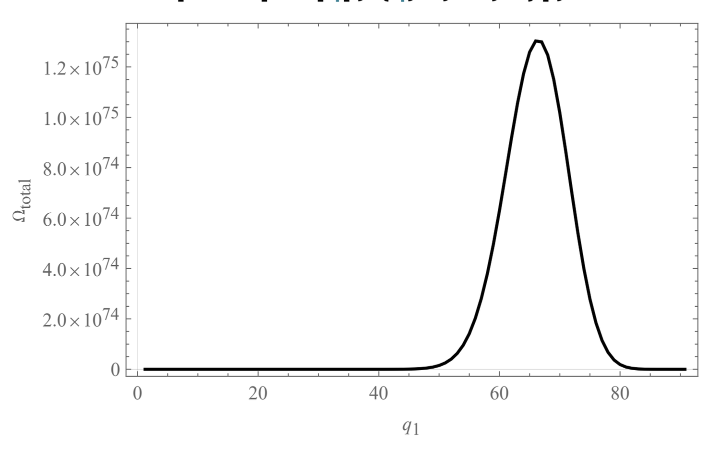
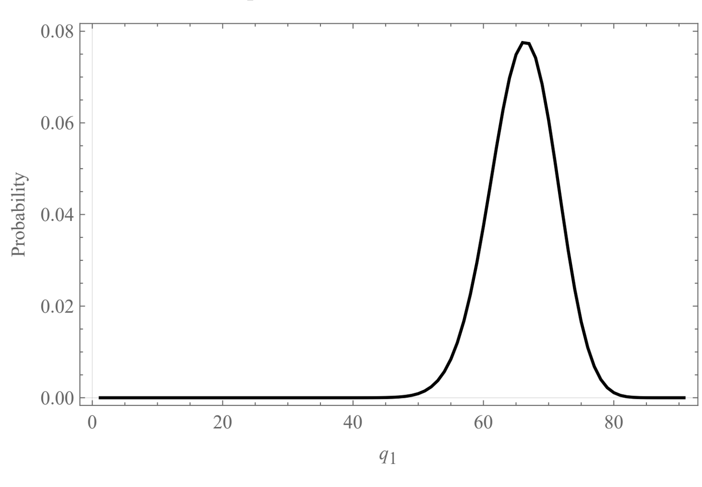
PHYS430 HW.3

Problem 1):

```
Mac[q_] := Binomial[q + 143 - 1, q] * Binomial[90 - q + 55 - 1, 90 - q];
max :=
  Reverse@{Table[Mac[q], {q, 0, 90, 1}] / Total[Table[Mac[q], {q, 0, 90, 1}]] // Max,
   Position[Table[Mac[q], {q, 0, 90, 1}] / Total[Table[Mac[q], {q, 0, 90, 1}]], Table[Mac[q], {q, 0, 90, 1}] / Total[Table[Mac[q], {q, 0, 90, 1}]] // Max][[1]][[1]] - 1};
min :=
  Reverse@{Table[Mac[q], {q, 0, 90, 1}] / Total[Table[Mac[q], {q, 0, 90, 1}]] // Min,
   Position[Table[Mac[q], {q, 0, 90, 1}] / Total[Table[Mac[q], {q, 0, 90, 1}]], Table[Mac[q], {q, 0, 90, 1}] / Total[Table[Mac[q], {q, 0, 90, 1}]] // Min][[1]][[1]] - 1};
ListLinePlot[Table[Mac[q], {q, 0, 90, 1}], PlotTheme → "Scientific", FrameLabel → {"q1", "Ωtotal"}, PlotStyle → Black]
```




```
ListLinePlot[Table[Mac[q], {q, 0, 90, 1}] / Total[Table[Mac[q], {q, 0, 90, 1}]], PlotTheme → "Scientific", FrameLabel → {"q1", "Probability"},
 PlotStyle → Black]
```




```
max // N
min // N
```

{65., 0.0775195}

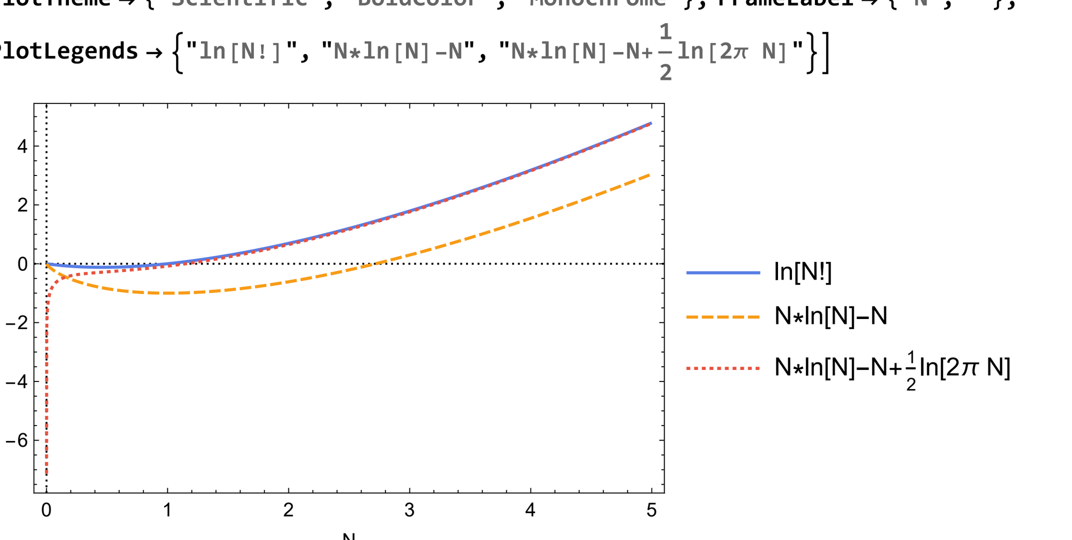
$\{0., 9.62927 \times 10^{-37}\}$

The most probable macrostate is at $q_1 = 65$, with a probability of 0.08. The least probable macrostate is ay q_1=0 with a probability of 9.6×10^{-37}

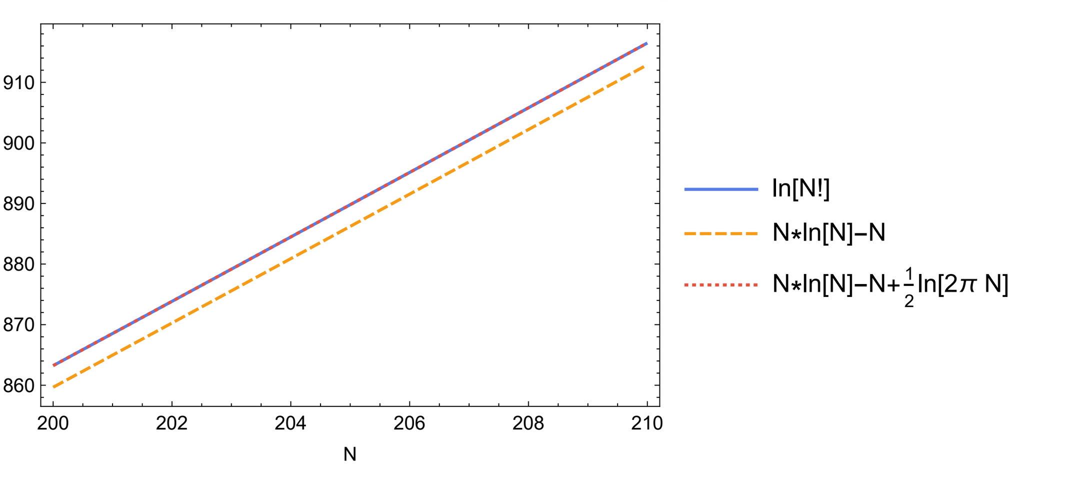
Problem 2):

The plots:

```
Plot[{Log[s!], s*Log[s] - s, s*Log[s] - s + 1/2 Log[2 π s]}, {s, 0, 5},
 PlotTheme → {"Scientific", "BoldColor", "Monochrome"}, FrameLabel → {"N", ""},
 PlotLegends → {"ln[N!]", "N*ln[N]-N", "N*ln[N]-N+1/2 ln[2π N]"}]
```




```
Plot[{Log[s!], s*Log[s] - s, s*Log[s] - s + 1/2 Log[2 π s]}, {s, 200, 210},
 PlotTheme → {"Scientific", "BoldColor", "Monochrome"}, FrameLabel → {"N", ""},
 PlotLegends → {"ln[N!]", "N*ln[N]-N", "N*ln[N]-N+1/2 ln[2π N]"}]
```



Percentage difference, which is defined as $\frac{|v1-v2|}{\left[\frac{v1+v2}{2}\right]} * 100\%$:

```
SetAttributes[Pdiff, HoldAll];
Pdiff[f1_, f2_] :=
 "The %Diff between " <> ToString[HoldForm[f1] // TraditionalForm] <> " and " <>
   ToString[HoldForm[f2] // TraditionalForm] <> " is: " <> ToString[Abs[f1 - f2] / ((f1+f2)/2) * 100 // N, TraditionalForm] <> "%" //
  TraditionalForm

n = 1*10^4;

Pdiff[Log[n!], n*Log[n] - n]
Pdiff[Log[n!], n*Log[n] - n + 1/2 Log[2 π n]]
Pdiff[n!, n*Exp[-n]] // Quiet
Pdiff[n!, n*Exp[-n] * √(2 π*n)] // Quiet
```

The %Diff between $\log(n!)$ and $n \log(n) - n$ is: 0.00672802%

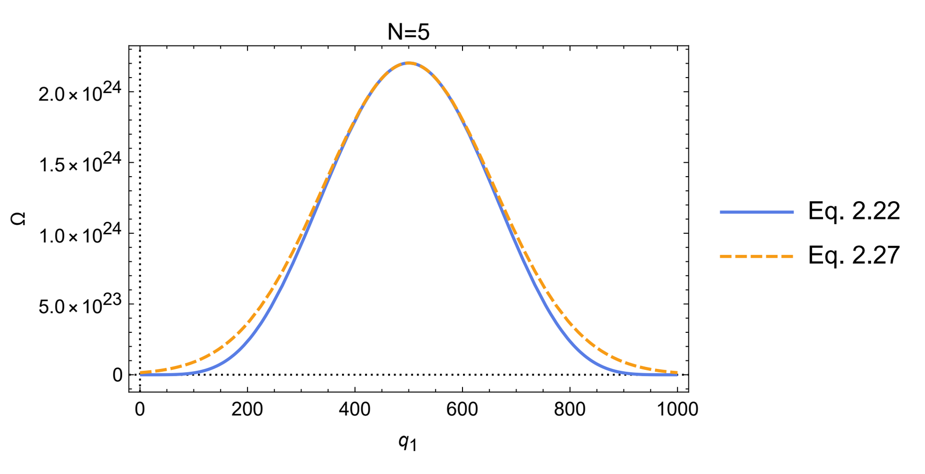
The %Diff between $\log(n!)$ and $n \log(n) - n + \frac{1}{2} \log(2 \pi n)$ is: 1.01491×10^{-8}%

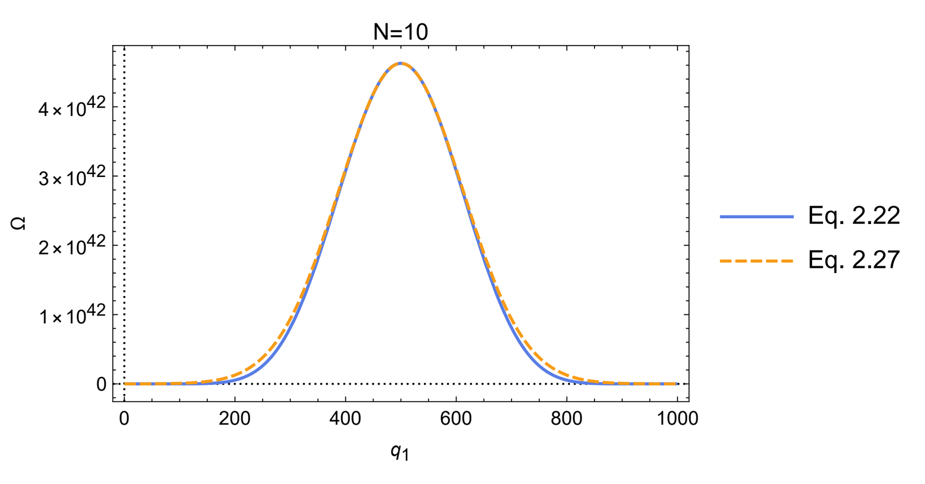
The %Diff between $n!$ and $n \exp(-n)$ is: 200.0000000000000%

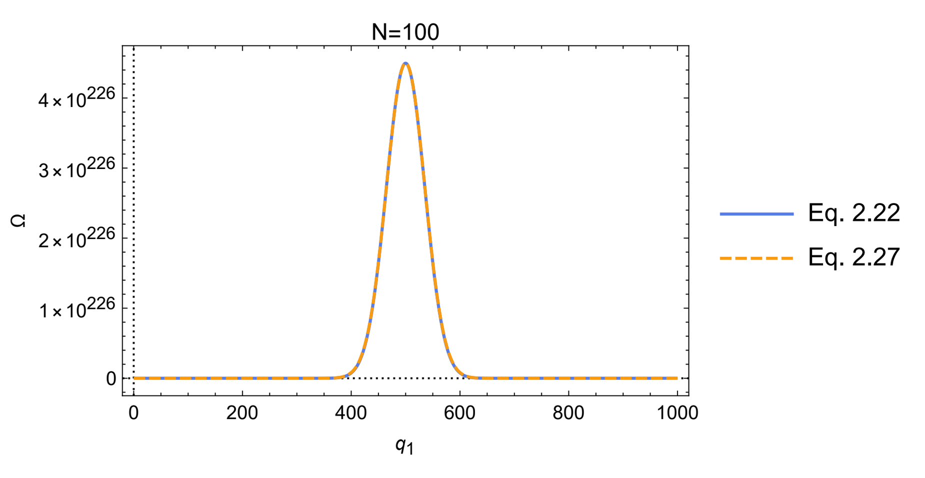
The %Diff between $n!$ and $n \exp(-n) \sqrt{2 \pi n}$ is: 200.0000000000000%

Problem 3):

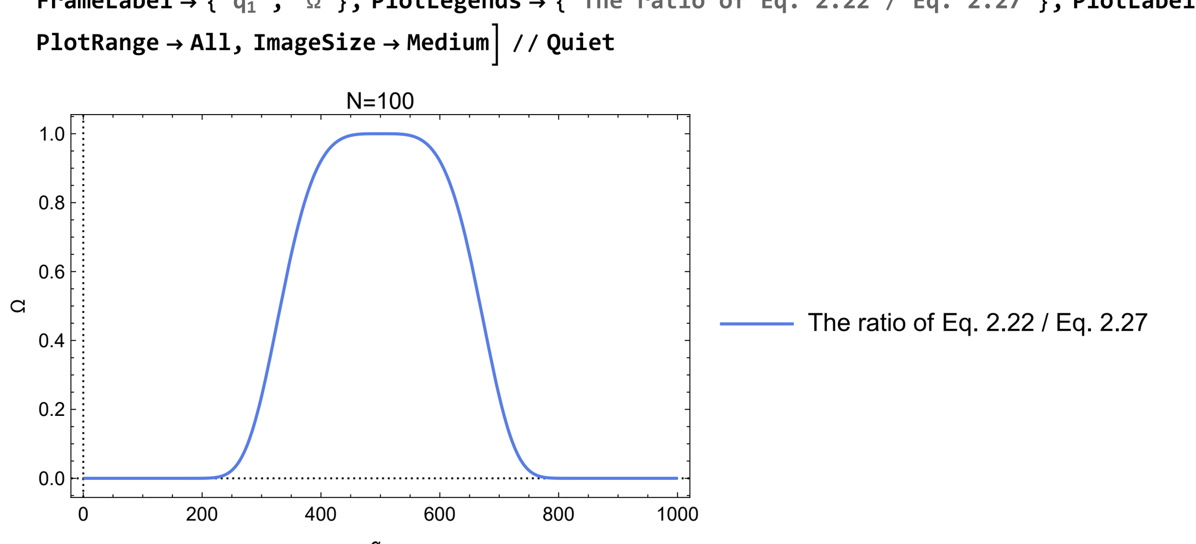
```
apmac[q_, s_, qm_] := (E/s)^(2 s) * (q * (qm - q))^s;
ap2mac[q_, s_, qm_] := (E/s)^(2 s) * (qm/2)^(2 s) * Exp[-s * ((2 z)/qm)^2] /. z → q - qm/2;

Plot[{apmac[q, 5, 1000], ap2mac[q, 5, 1000]}, {q, 0, 1000}, PlotTheme → {"Scientific", "BoldColor", "Monochrome"},
 FrameLabel → {"q1", "Ω"}, PlotLegends → {"Eq. 2.22", "Eq. 2.27"}, PlotLabel → "N=5", ImageSize → Medium]
Plot[{apmac[q, 10, 1000], ap2mac[q, 10, 1000]}, {q, 0, 1000}, PlotTheme → {"Scientific", "BoldColor", "Monochrome"},
 FrameLabel → {"q1", "Ω"}, PlotLegends → {"Eq. 2.22", "Eq. 2.27"}, PlotLabel → "N=10", ImageSize → Medium]
Plot[{apmac[q, 100, 1000], ap2mac[q, 100, 1000]}, {q, 0, 1000}, PlotTheme → {"Scientific", "BoldColor", "Monochrome"},
  FrameLabel → {"q1", "Ω"}, PlotLegends → {"Eq. 2.22", "Eq. 2.27"}, PlotLabel → "N=100", PlotRange → All,
  ImageSize → Medium] // Quiet
```



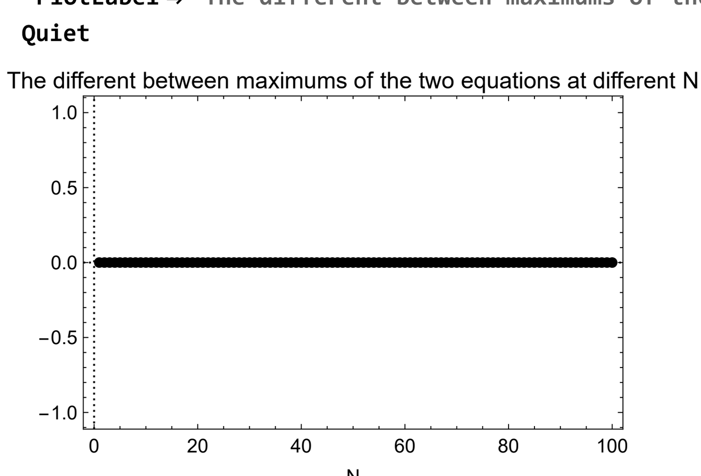





```
Plot[Max@apmac[q, 100, 1000] / Max@ap2mac[q, 100, 1000], {q, 0, 1000}, PlotTheme → {"Scientific", "BoldColor", "Monochrome"},
  FrameLabel → {"q1", "Ω"}, PlotLegends → {"The ratio of Eq. 2.22 / Eq. 2.27"}, PlotLabel → "N=100",
  PlotRange → All, ImageSize → Medium] // Quiet
```




```
ListLinePlot[Table[Max@Table[apmac[q, s, 1000] // N, {q, 0, 1000}] - Max@Table[ap2mac[q, s, 1000], {q, 0, 1000}] // N,
   {s, 1, 100, 1}], PlotRange → All, PlotTheme → {"Scientific", "BoldColor", "Monochrome"},
  PlotLabel → "The different between maximums of the two equations at different N", FrameLabel → {"N", " "}] //
 Quiet
```

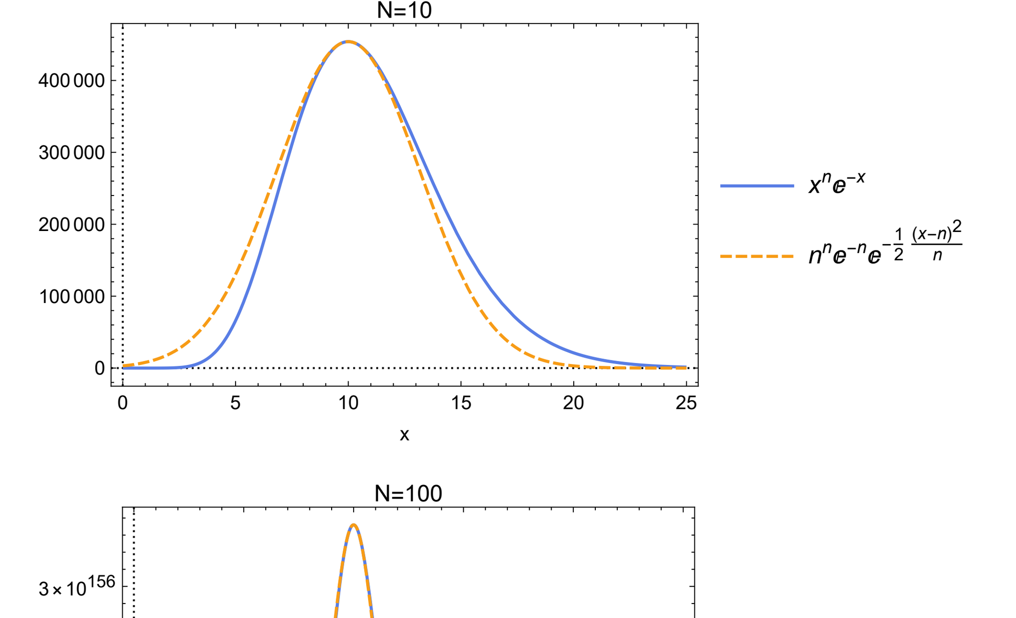


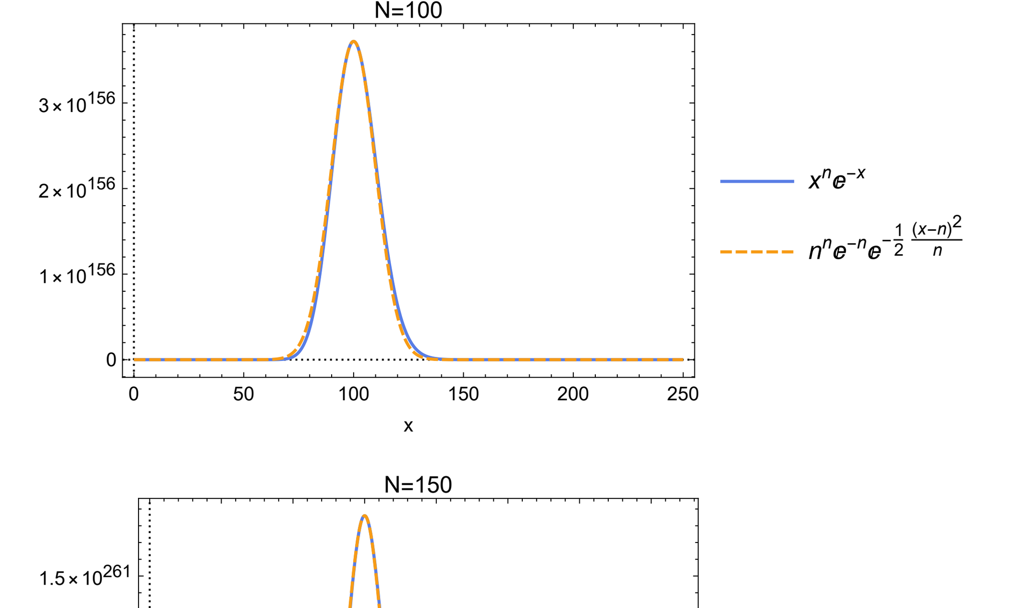
The approximation is more accurate when N is Large. Moreover, the ratio is 1 around the maximum, which is the place where the two approximations are almost equal. Finally, the maximum of both plots is always the same up to N=100 which I checked. The approximation is valid.

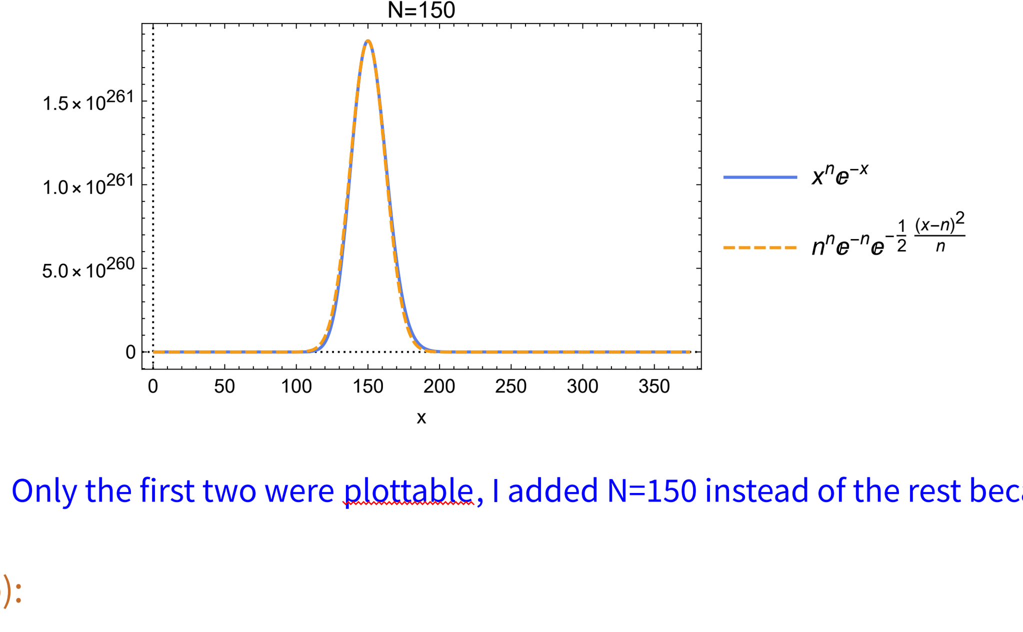
Problem 4):

a):

```
int1[x_, n_] := x^n Exp[-x];
int2[x_, n_] := n^n Exp[-n] Exp[-1/2 (x - n)^2/n];

Plot[{int1[x, 10.], int2[x, 10.]}, {x, 0, 25}, PlotTheme → {"Scientific", "BoldColor", "Monochrome"},
  FrameLabel → {"x", " "}, PlotLegends → {"x^n e^-x", "n^n e^-n e^(-1/2 (x-n)^2/n)"}, PlotLabel → "N=10", PlotRange → All,
  ImageSize → Medium, PlotRange → All] // Quiet
Plot[{int1[x, 100.], int2[x, 100.]}, {x, 0, 250}, PlotTheme → {"Scientific", "BoldColor", "Monochrome"},
  FrameLabel → {"x", " "}, PlotLegends → {"x^n e^-x", "n^n e^-n e^(-1/2 (x-n)^2/n)"}, PlotLabel → "N=100", PlotRange → All,
  ImageSize → Medium, PlotRange → All] // Quiet
Plot[{int1[x, 150], int2[x, 150]}, {x, 0, 375}, PlotTheme → {"Scientific", "BoldColor", "Monochrome"},
  FrameLabel → {"x", " "}, PlotLegends → {"x^n e^-x", "n^n e^-n e^(-1/2 (x-n)^2/n)"}, PlotLabel → "N=150", PlotRange → All,
  ImageSize → Medium] // Quiet
```



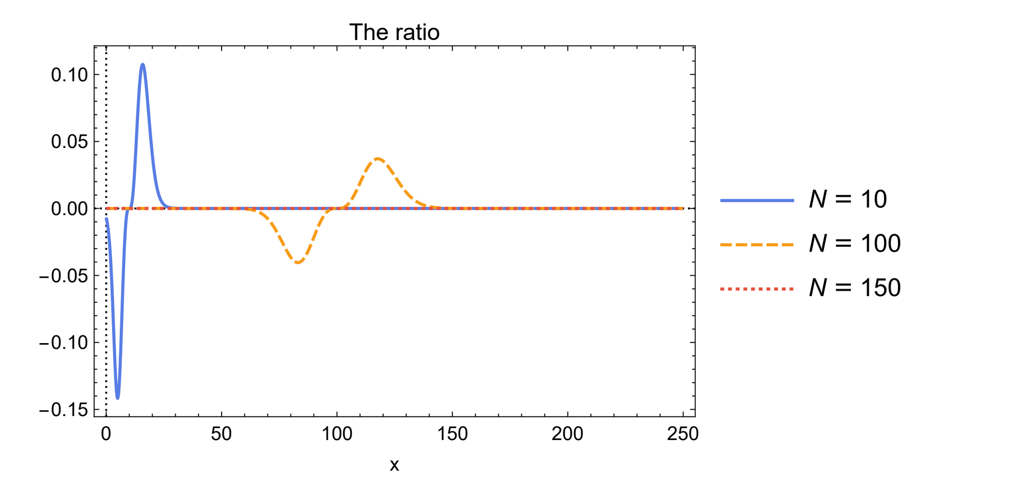




Only the first two were plottable, I added N=150 instead of the rest because it is the last one I can plot.

b):

```
sa = {(int1[x, z] - int2[x, z]) / (z^z e^-z) /. z → 10, (int1[x, z] - int2[x, z]) / (z^z e^-z) /. z → 100, (int1[x, z] - int2[x, z]) / (z^z e^-z) /. z → 150};

Plot[sa, {x, 0, 250}, PlotTheme → {"Scientific", "BoldColor", "Monochrome"}, FrameLabel → {"x", " "},
  PlotLegends → {TraditionalForm[N == 10], TraditionalForm[N == 100], TraditionalForm[N == 150]},
  PlotLabel → "The ratio", PlotRange → All, ImageSize → Medium] // Quiet
```



The difference is negligible when N is big, which means this approximation is valid for big systems.