Q 1:

$$\Omega \propto AA_r \implies \frac{L}{\Delta x} \cdot \frac{L_r}{\Delta r_x} = \frac{LL_r}{h} = \Omega_x \implies \Omega_1 = \Omega_x \Omega_y = \frac{AA_r}{h^2} \implies \Omega_N = \frac{A^N A_r}{N! h^{2N}}$$

$$\Omega_N = \frac{1}{N!} \frac{A^N}{h^{2n}} \frac{2\pi^N}{(N-1)!} \sqrt{2mU}^{2N-1} \approx \frac{A^N \pi^N (emU)^{2N}}{N!} = \frac{(2\pi mAU)^N}{N!N!h^{2N}}$$

Now, we use $S = k \ln \Omega$:

$$S = k \ln \Omega = k \left\{ N \ln(2\pi mAU) - 2 \ln N! - 2N \ln h \right\} = kN \left\{ \ln \left[\frac{2\pi mAU}{N^2 h^2} \right] + 2 \right\}$$

Q 2:

a):

$$S = k \ln \Omega = k \ln \left[\frac{N!}{(N-n)!n!} \right] = k \left\{ \ln N! - \ln(N-n)! + n \ln(N-n) - n \ln n \right\}$$

$$= k \left\{ N \ln N - N \left(\ln N + \ln \left(1 - \frac{n}{N} \right) \right) + n \left(\ln N + \ln \left(1 - \frac{n}{N} \right) \right) - n \ln n \right\}$$

$$\ln(1+\epsilon) = \epsilon$$

$$= k \left\{ n + n \ln N - \frac{n^2}{N} - n \ln n \right\} = kn \left\{ 1 + \ln \frac{N}{n} \right\}$$

Q 3:

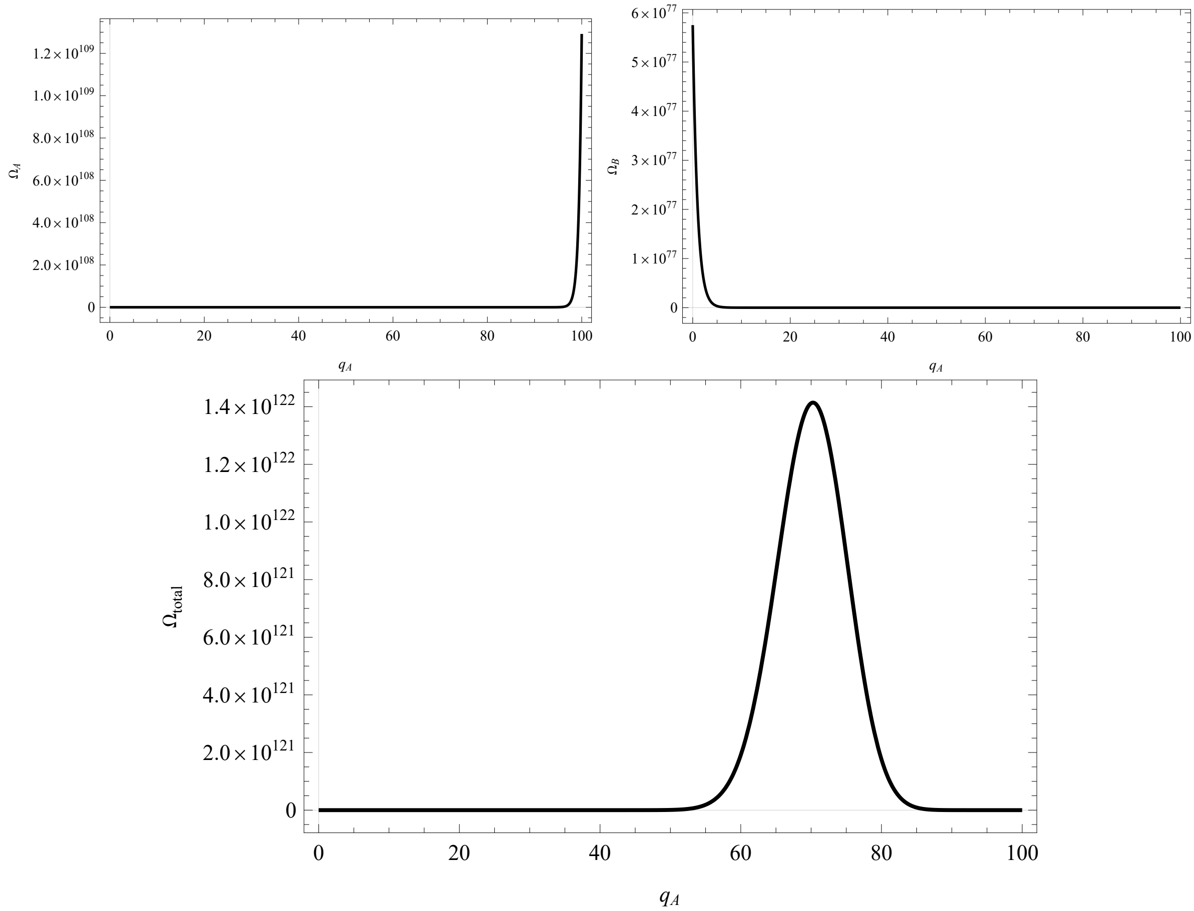

Figure 1: The multiplicity of system A, B, both, respectively, as a function of energy units of system A.

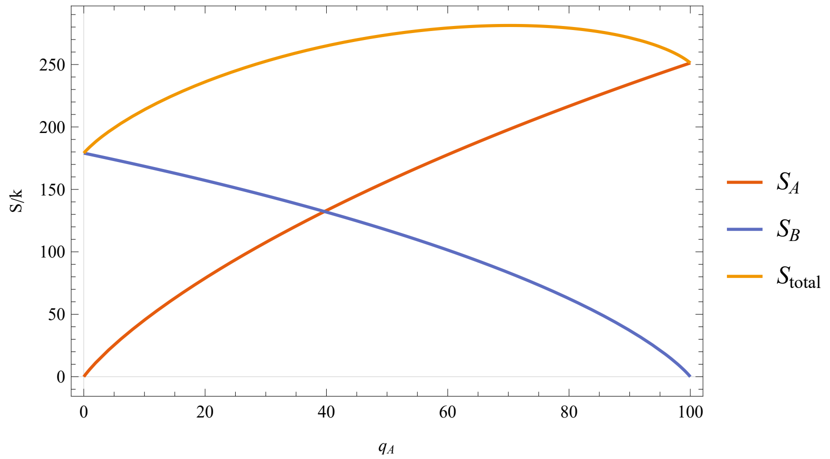

Figure 2: The entropy of system A, B, both, respectively, as a function of energy units of system A. The maximum occurs at $q_A = 70$.

Q 4:

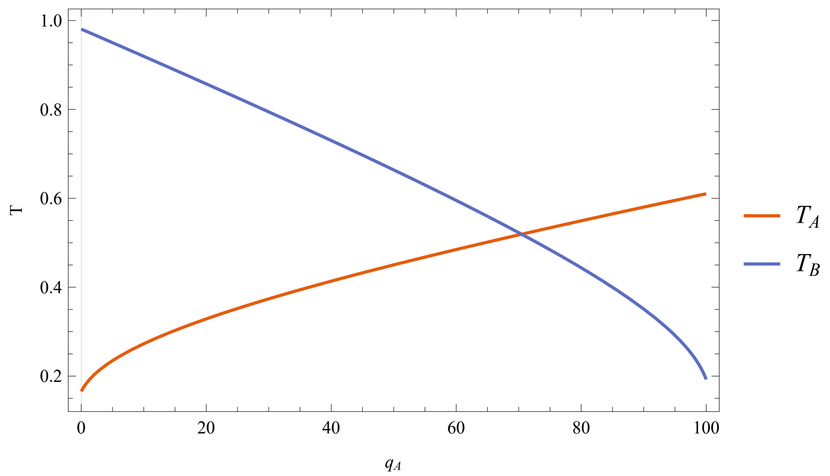

Figure 3: The temperature of system A, and B as a function of energy units of system A. At the maximum of the entropy $q_A = 70$, both system have the same temperature. Implying that the system reaches thermal equilibrium while maximizing its entropy

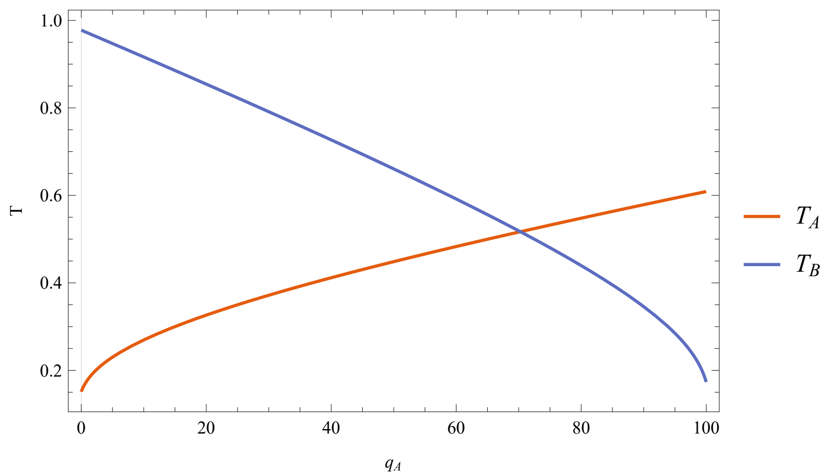

Figure 4: Same as Figure 3, but done analytically via Mathematica