Q 1:

$$\frac{G_F}{\hbar^3 c^3} = \frac{\sqrt{2}}{8}\frac{g_W^2}{M_W^2 c^4}$$

Solving for M_W:

$$M_W c^2 = \left(\frac{\sqrt{2} g_W^2 \hbar^3 c^3}{8 G_F}\right)^{1/2}$$

Using Sol manual

$$g_w = \frac{g_e}{\sin\theta_w}; \quad g_e = \sqrt{4\pi\alpha}$$

$$\therefore M_W c^2 = 77.5\ GeV$$

$$M_Z = \frac{M_W}{\cos\theta_w} = 88.4\ GeV$$

%Error to experiment:

$$M_W c^2 \implies 3.59\%; \quad M_Z c^2 \implies 3.05\%$$

Q 2:

Sol. Man.
a):

From example 9.4, while switching $1 \leftrightarrow 3$:

$$\mathcal{M} = \frac{g_z^2}{8 M_Z c^2}\left[\bar{v}(1)\gamma^\mu(1-\gamma^5)v(3)\right]\left[\bar{u}(4)\gamma_\mu(c_V - c_A\gamma^5)u(2)\right]$$

$$(c_V + c_A)^2 \leftrightarrow (c_V - c_A)^2$$

Now Eqs 9.99 and 9.100:

$$\frac{d\sigma}{d\Omega} = 2\left(\frac{\hbar c}{\pi}\right)^2\left(\frac{g_z}{4 M_Z c^2}\right)^4 E^2\left[(c_V - c_A)^2 + (c_V + c_A)^2\cos^4\frac{\theta}{2}\right]$$

$$\sigma = \frac{2}{3\pi}(\hbar c)^2\left(\frac{g_z}{2 M_Z c^2}\right)^4 E^2\left[(c_V^2 + c_A^2 - c_v c_A)\right]$$

b):

$$R \equiv \frac{c_V^2 + c_A^2 - c_V c_A}{c_V^2 + c_A^2 + c_V c_A}$$

Using Table 9.1 for $c_V \& c_A$ for neutrinos:

$$\therefore R = \frac{0.2514 - 0.0186}{0.2514 + 0.0186} = 0.862$$

Q 3:

$$\mathcal{L} = \left[i\hbar c\bar{\psi}\gamma^\mu\partial_\mu - mc^2\bar{\psi}\psi\right] - \left(q\bar{\psi}\gamma^\mu\psi\right)A_\mu$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})} = 0; \quad \frac{\partial\mathcal{L}}{\partial\bar{\psi}} = i\hbar c\gamma^\mu\partial_\mu - mc^2\psi - q\gamma^\mu\psi A_\mu$$

$$\therefore i\hbar c\gamma^\mu(\partial_\mu\psi) - mc\psi = q\gamma^\mu\psi A_\mu$$

Similarly for ψ:

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)} = i\hbar c\bar{\psi}\gamma^\mu; \quad \frac{\partial\mathcal{L}}{\partial\psi} = -mc^2\bar{\psi} - \frac{q}{c}\bar{\psi}\gamma^\mu A_\mu$$

$$\therefore i\hbar c(\partial_\mu\bar{\psi})\gamma^\mu + mc\bar{\psi} = -\frac{q}{c}\bar{\psi}\gamma^\mu A_\mu$$

Q 4:

a):

$$\delta\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\phi_i}\delta\phi_i + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta(\partial_\mu\phi_i)$$

Applying Euler-Lagrange:

$$\delta\mathcal{L} = \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\right)\delta\phi_i + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\partial_\mu(\delta\phi_i) = \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}\delta\phi_i\right)$$