Q 1:

$$P^{\mu}_{A} + P^{\mu}_{B} = P^{\mu}_{C} + P^{\mu}_{D}$$
$$Transform\ using\ \Lambda^{\nu}_{\mu}$$
$$\Lambda^{\nu}_{\mu}(P^{\mu}_{A} + P^{\mu}_{B}) = \Lambda^{\nu}_{\mu}(P^{\mu}_{C} + P^{\mu}_{D})$$
$$P^{\nu}_{A} + P^{\nu}_{B} = P^{\nu}_{C} + P^{\nu}_{D}$$

Which is also conserved.

Q 2:

a) $p + p \rightarrow p + p + \pi^0$

b) $p + p \rightarrow p + p + \pi^+ + \pi^-$

c) $\pi^- + p \rightarrow p + \bar{p} + n$

d) $\pi^- + p \rightarrow K^0 + \Sigma^0$

e) $p + p \rightarrow p + \Sigma^+ + K^0$

Problem 3.16 result:

$$E_{min} = \frac{M^2 - m_A^2 - m_B^2}{2m_B}c^2; \qquad M = m_1 + m_2 + \cdots + m_n$$

a):

$$E_{min} = \frac{(2m_p + m_{\pi^0})^2 - 2m_p^2}{2m_p}c^2 = 1218\ MeV$$

b):

$$E_{min} = \frac{(2m_p + m_{\pi^+} + m_{\pi^-})^2 - 2m_p^2}{2m_p}c^2 = 1538\ MeV$$

c):

$$E_{min} = \frac{(2m_p + m_n)^2 - m_{\pi^-}^2 - m_p^2}{2m_p}c^2 = 3747\ MeV$$

d):

$$E_{min} = \frac{(m_{K^0} + m_{\Sigma^0})^2 - m_{\pi^-}^2 - m_p^2}{2m_p}c^2 = 1043\ MeV$$

e):

$$E_{min} = \frac{(2m_p + m_{K^0} + m_{\Sigma^+})^2 - 2m_p^2}{2m_p}c^2 = 21734\ MeV$$

Q 3:

a) $\pi^- \to \mu^- + \bar{\nu}_\mu$

b) $\pi^0 \to \gamma + \gamma$

c) $K^+ \to \pi^+ + \pi^0$

d) $\Lambda \to p + \pi^-$

e) $\Omega^- \to \Lambda + K^-$

Problem 3.19 result:

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A}c^2; \qquad E_C = \frac{m_A^2 + m_C^2 - m_B^2}{2m_A}c^2$$

a):

$$E_{\mu^-} = \frac{m_{\pi^-}^2 + m_{\mu^-}^2 - m_{\bar{\nu}_\mu}^2}{2m_{\pi^-}}c^2 = 109.8\ MeV; \qquad E_{\bar{\nu}_\mu} = \frac{m_{\pi^-}^2 + m_{\bar{\nu}_\mu}^2 - m_{\mu^-}^2}{2m_{\pi^-}}c^2 = 29.8\ MeV$$

b):

$$E_\gamma = \frac{m_{\pi^0}^2 + m_\gamma^2 - m_\gamma^2}{2m_{\pi^0}}c^2 = 67.5\ MeV$$

c):

$$E_{\pi^+} = \frac{m_{K^+}^2 + m_{\pi^+}^2 - m_{\pi^0}^2}{2m_{K^+}}c^2 = 248.1\ MeV; \qquad E_{\pi^0} = \frac{m_{K^+}^2 + m_{\pi^0}^2 - m_{\pi^+}^2}{2m_{K^+}}c^2 = 245.6\ MeV$$

d):

$$E_p = \frac{m_\Lambda^2 + m_p^2 - m_{\pi^-}^2}{2m_\Lambda}c^2 = 943.6\ MeV; \qquad E_{\pi^-} = \frac{m_\Lambda^2 + m_{\pi^-}^2 - m_p^2}{2m_\Lambda}c^2 = 172.0\ MeV$$

e):

$$E_\Lambda = \frac{m_{\Omega^-}^2 + m_\Lambda^2 - m_{K^-}^2}{2m_{\Omega^-}}c^2 = 1136\ MeV; \qquad E_{K^-} = \frac{m_{\Omega^-}^2 + m_{K^-}^2 - m_\Lambda^2}{2m_{\Omega^-}}c^2 = 537\ MeV$$

Q 4 :

First, we need to find the velocity of the muon using:

$$P_\mu = -P_\nu; \qquad P_\mu, P_\nu \ are\ 4-momentum\ vector$$

After working out the 4-momentum conservation, we will find this value of the muon velocity (Example 3.3):

$$v = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}c = 0.27c$$

$$\gamma = \frac{1}{\sqrt{1-0.27^2}} = 1.039$$

$$d = \gamma v \tau = 1.039 \times (0.27 \times 3 \times 10^8) \times (2.20 \times 10^{-6}) = 184\ m$$

Q 5:

$$s = \frac{(P_A + P_B)^2}{c^2} = \frac{(\frac{E_A+E_B}{c})^2 - (p_A + p_B)^2}{c^2}$$

Since $p_A = -p_B$ & $E_A = E_B = E$ and $E = \sqrt{p^2c^2 + m^2c^4}$:

$$s = \frac{\frac{4E^2}{c^2} - 0}{c^2} = \frac{4(p^2c^2 + m^2c^4)}{c^4} = \frac{4(p^2 + m^2c^2)}{c^2} \checkmark$$

$$t = \frac{(P_A - P_C)^2}{c^2} = \frac{(\frac{E_A-E_C}{c})^2 - (p_A - p_C)^2}{c^2}$$

Since $E_A = E_C = E$ and $p_A^2 = p_C^2 = p^2$:

$$t = \frac{(\frac{E-E}{c})^2 - (p_A - p_C)^2}{c^2} = -\frac{p^2 + p^2 - 2p_A \cdot p_C}{c^2} = -\frac{2p^2(1-\cos\theta)}{c^2} \checkmark$$

For u, same as t excepts the cosine of the angle between p_A and p_D will yield negative that of p_A and p_c, so:

$$u = \frac{(\frac{E-E}{c})^2 - (p_A - p_D)^2}{c^2} = -\frac{p^2 + p^2 - 2p_A \cdot p_D}{c^2} = -\frac{2p^2(1+\cos\theta)}{c^2} \checkmark$$

$$m_{\pi^0} = 134.98\ MeV/c^2 \qquad m_{\pi^\pm} = 139.57\ MeV/c^2$$

$$m_{\Sigma^0} = 1192.64\ MeV/c^2 \qquad m_{\Sigma^+} = 1189.37\ MeV/c^2$$

$$m_{K^0} = 497.61\ MeV/c^2 \qquad m_{K^\pm} = 493.67\ MeV/c^2$$

$$m_{\Omega^-} = 1672.45\ MeV/c^2 \qquad m_\Lambda = 1115.68\ MeV/c^2$$