Firstly, this is my Mathematica code that I used for part 1&3:

```
σ₊ = {{0, 1}, {0, 0}}; σ₋ = {{0, 0}, {1, 0}};
c†j_ := KroneckerProduct@@N@ReplacePart[ReplacePart[Table[IdentityMatrix[2], 6], Table[{j-i}, {i, j-1}] → -PauliMatrix[3]], j → σ₊];
cj_ := KroneckerProduct@@N@ReplacePart[ReplacePart[Table[IdentityMatrix[2], 6], Table[{j-i}, {i, j-1}] → -PauliMatrix[3]], j → σ₋]
ListPlot[Sort@Eigenvalues[1 (Sum[(c†i.ci+1), {i, 1, 5}] + c†6.c1) + 2 (Sum[(c†i.c†i+1.ci.ci+1), {i, 1, 5}] + c†6.c†1.c6.c1)], PlotStyle → Black, PlotMarkers → {Automatic, 3}]
Grid[{{c5.c†5 + c†5.c5 // SparseArray, c5.c†6 + c†6.c5 // SparseArray}, {c5.c6 + c6.c5 // SparseArray, c†5.c†6 + c†6.c†5 // SparseArray}}]
```

Figure 1: Mathematica code used for parts 1&3

1 Proving Fermonic Anti-Commutator Relations

For this part I did not do it analytically, I did it numerically. I set up a system with 7 particles, and defined the creation and destruction operators, then I applied the anti-commutator relations for two consecutive operators, i, $i+1$. I found that the anti-commutator relations holds for this definition of operators, as seen in Figure. 2:

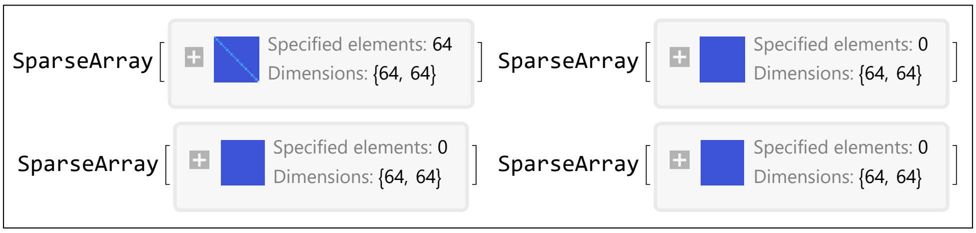

Figure 2: Results for $\{c_5, c_5^\dagger\}$; $\{c_5, c_6^\dagger\}$; $\{c_5, c_6\}$; $\{c_5^\dagger, c_6^\dagger\}$

2 Plotting Eigenvalues for a Hamiltonian

$$H = t\sum_{i}^{6} c_i^\dagger c_{i+1} + U\sum_{i}^{6} c_i^\dagger c_{i+1}^\dagger c_i c_{i+1}$$

Using the same system, these are the eigenvalues for above Hamiltonian:

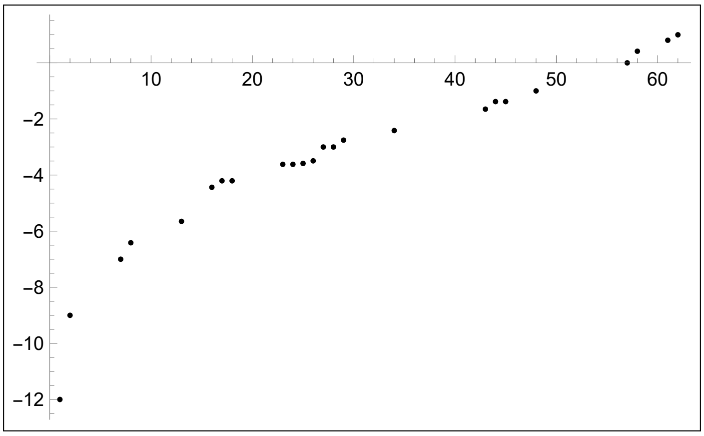

Figure 3: Sorted Eigenvalues of the Hamiltonian